

Chapter 12. Distance and Section Formulae

Ex 12.1

Answer 2.

Coordinates of origin are P (0, 0).

(a) P(0, 0), Q(5, 12)

$$PQ = \sqrt{(12 - 0)^2 + (5 - 0)^2} = \sqrt{144 + 25} = \sqrt{169} = 13 \text{ units}$$

(b) P(0, 0), Q(6, 8)

$$PQ = \sqrt{(6 - 0)^2 + (8 - 0)^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ units.}$$

(c) P(0, 0), Q(8, 15)

$$PQ = \sqrt{(8 - 0)^2 + (15 - 0)^2} = \sqrt{64 + 225} = \sqrt{289} = 17 \text{ units.}$$

(d) P(0, 0), Q(0, 11)

$$PQ = \sqrt{(0 - 0)^2 + (11 - 0)^2} = \sqrt{121} = 11 \text{ units}$$

(e) P(0, 0), Q(13, 0)

$$PQ = \sqrt{(13 - 0)^2 + (0 - 0)^2} = \sqrt{169} = 13 \text{ units}$$

Answer 3.

$$(a) A(p+q, p-q), B(p-q, p-q)$$

$$AB = \sqrt{(p-q-p)^2 + (p-q-p+q)^2}$$

$$= \sqrt{4q^2 + 0} = 2q \text{ units}$$

$$(b) A(\sin\theta, \cos\theta), B(\cos\theta, -\sin\theta)$$

$$AB = \sqrt{(\cos\theta - \sin\theta)^2 + (-\sin\theta - \cos\theta)^2}$$

$$= \sqrt{\cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta + \sin^2\theta + \cos^2\theta + 2\cos\theta\sin\theta}$$

$$= \sqrt{2} \text{ units.}$$

$$(c) A(\sec\theta, \tan\theta), B(-\tan\theta, \sec\theta)$$

$$AB = \sqrt{(-\tan\theta - \sec\theta)^2 + (\sec\theta - \tan\theta)^2}$$

$$= \sqrt{\tan^2\theta + \sec^2\theta + 2\tan\theta\sec\theta + \sec^2\theta + \tan^2\theta - 2\tan\theta\sec\theta}$$

$$= \sqrt{2\sec^2\theta + 2\tan^2\theta} \text{ units.}$$

$$i) A(\sin\theta - \operatorname{cosec}\theta, \cos\theta - \cot\theta)$$

$$B(\cos\theta - \operatorname{cosec}\theta, -\sin\theta - \cot\theta)$$

$$AB = \sqrt{(\cos\theta - \operatorname{cosec}\theta - \sin\theta + \operatorname{cosec}\theta)^2 + (-\sin\theta - \cot\theta - \cos\theta + \cot\theta)^2}$$

$$= \sqrt{(\cos\theta - \sin\theta)^2 + (-\sin\theta - \cos\theta)^2}$$

$$= \sqrt{\cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta + \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}$$

$$= \sqrt{2} \text{ units.}$$

Answer 4.

Let the point on x-axis be $(x, 0)$ given abscissa is -5.

\therefore point is $P(-5, 0)$

Let $(7, 5)$ be point A

$$AP = \sqrt{(7+5)^2 + (5-0)^2}$$

$$= \sqrt{144 + 25}$$

$$= \sqrt{169}$$

$$= 13 \text{ units}$$

Answer 5.

Point on the line $y = 0$ lies on x-axis given abscissa is 1.

\therefore point is $P(1, 0)$

Let $(13, -9)$ be point A

$$\begin{aligned} AP &= \sqrt{(13-1)^2 + (-9-0)^2} \\ &= \sqrt{144 + 81} \\ &= \sqrt{225} \\ &= 15 \text{ units} \end{aligned}$$

Answer 6.

Point on the line $x = 0$ lies on given its ordinate is 9.

\therefore point is $P(0, 9)$

Let the point $(12, 5)$ be A.

$$\begin{aligned} AP &= \sqrt{(12-0)^2 + (5-9)^2} \\ &= \sqrt{144 + 16} \\ &= \sqrt{160} \\ &= 4\sqrt{10} \text{ units.} \end{aligned}$$

Answer 7.

Let the points $(5, a)$ and $(1, -5)$ be P and Q respectively.

Given, $PQ = 5$ units

$$\sqrt{(5-1)^2 + (a+5)^2} = 5$$

squaring both sides, we get,

$$16 + a^2 + 25 + 10a = 25$$

$$\Rightarrow a^2 + 10a + 16 = 0$$

$$\Rightarrow a^2 + 8a + 2a + 16 = 0$$

$$\Rightarrow (a+8)(a+2) = 0$$

$$\therefore a = -8, -2$$

Answer 8.

Let the points $(m, -4)$ and $(3, 2)$ be A and B respectively.

Given $AB = 3\sqrt{5}$ units

$$\sqrt{(m-3)^2 + (-4-2)^2} = 3\sqrt{5}$$

squaring both sides

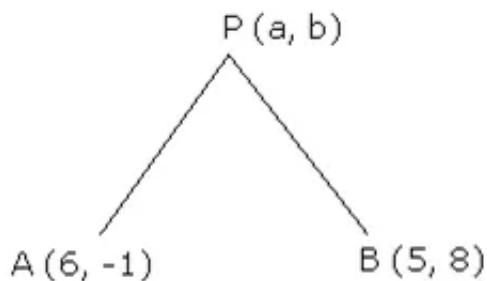
$$m^2 - 6m + 9 + 36 = 45$$

$$\Rightarrow m^2 - 6m = 0$$

$$\Rightarrow m(m-6) = 0$$

$$\Rightarrow m = 0 \text{ or } 6.$$

Answer 9.



Given, $PA = PB$

$$\therefore PA^2 = PB^2$$

$$\Rightarrow (a-6)^2 + (b+1)^2 = (a-5)^2 + (b-8)^2$$

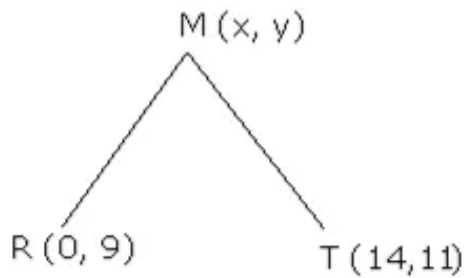
$$\Rightarrow a^2 + 36 - 12a + b^2 + 1 + 2b = a^2 + 25 - 10a + b^2 + 64 - 16b$$

$$\Rightarrow -2a + 18b - 52 = 0$$

$$\Rightarrow -a + 9b - 26 = 0$$

$$\Rightarrow a = 9b - 26$$

Answer 10.



Given: $MR = MT$

$$\therefore MR^2 = MT^2$$

$$(x - 0)^2 + (y - 9)^2 = (x - 14)^2 + (y - 11)^2$$

$$x^2 + y^2 + 81 - 18y = x^2 + 196 - 28x + y^2 + 121 - 22y$$

$$81 - 18y = 196 - 28x + 121 - 22y$$

$$28x - 18y + 22y = 196 + 121 - 81$$

$$28x + 4y = 236$$

$$7x + y - 58 = 0$$

Answer 11.

P lies on y-axis and has ordinate

$$\therefore P(0, 5)$$

Q lies on x-axis and has an abscissa

$$\therefore Q(12, 0)$$

$$\therefore PQ = \sqrt{(12 - 0)^2 + (0 - 5)^2}$$

$$= \sqrt{144 + 25}$$

$$= \sqrt{169}$$

$$= 13 \text{ units.}$$

Answer 12.

P lies on x-axis and Q lies on y-axis

Let abscissa of P be x then ordinate of Q is $x-1$.

$$\therefore P(x, 0), \quad Q(0, x-1)$$

Given $PQ=5$ units

$$\sqrt{(x-0)^2 + (0-x+1)^2} = 5$$

squaring both sides

$$x^2 + x^2 + 1 - 2x = 25$$

$$2x^2 - 2x - 24 = 0$$

$$x^2 - x - 12 = 0$$

$$x^2 - 4x + 3x - 12 = 0$$

$$(x-4)(x+3) = 0$$

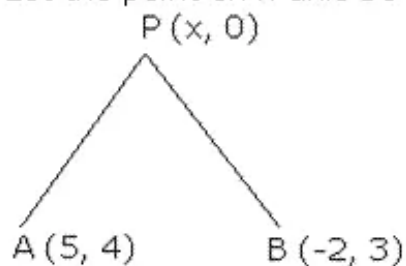
$$x = +4 \text{ or } -3$$

Coordinates of P are $(4, 0)$ or $(-3, 0)$

Coordinates of Q are $(0, 3)$ or $(0, -4)$.

Answer 13.

Let the point on x-axis be $P(x, 0)$.



Given,

$$PA = PB$$

$$PA^2 = PB^2$$

$$(x-5)^2 + (0-4)^2 = (x+2)^2 + (0-3)^2$$

$$x^2 + 25 - 10x + 16 = x^2 + 4 + 4x + 9$$

$$\Rightarrow -14x + 28 = 0$$

$$\Rightarrow 14x = 28$$

$$\Rightarrow x = 2$$

\therefore The point on x-axis is $(2, 0)$

Answer 14.

A (-4, 3). Let the other point B (x, 9).

Given, AB = 10 units

$$\sqrt{(-4-x)^2 + (3-9)^2} = 10$$

squaring both sides,

$$\Rightarrow 16 + x^2 + 8x + 36 = 100$$

$$\Rightarrow x^2 + 8x - 48 = 0$$

$$\Rightarrow x^2 + 12x - 4x - 48 = 0$$

$$\Rightarrow x(x+12) - 4(x+12) = 0$$

$$\Rightarrow (x-4)(x+12) = 0$$

$$\Rightarrow x = 4 \text{ or } -12$$

The abscissa of other ends is 4 or -12.

Answer 15.

A (5, 5), B (3, 4), C (-7, -1)

$$AB = \sqrt{(5-3)^2 + (5-4)^2} = \sqrt{4+1} = \sqrt{5} \text{ units}$$

$$BC = \sqrt{(3+7)^2 + (4+1)^2} = \sqrt{100+25} = 5\sqrt{5} \text{ units}$$

$$AC = \sqrt{(5+7)^2 + (5+1)^2} = \sqrt{144+36} = 6\sqrt{5} \text{ units}$$

$$AB + BC = \sqrt{5} + 5\sqrt{5} = 6\sqrt{5} = AC$$

$$\therefore AB + BC = AC$$

\therefore A, B and C are collinear points

P(5, 1), Q(3, 2), R(1, 3)

$$PQ = \sqrt{(5-3)^2 + (1-2)^2} = \sqrt{4+1} = \sqrt{5} \text{ units}$$

$$QR = \sqrt{(3-1)^2 + (2-3)^2} = \sqrt{4+1} = \sqrt{5} \text{ units}$$

$$PR = \sqrt{(5-1)^2 + (1-3)^2} = \sqrt{16+4} = \sqrt{20} \text{ units}$$

$$PQ + QR = \sqrt{5} + \sqrt{5} = 2\sqrt{5} = PR$$

$$\therefore PQ + QR = PR$$

\therefore P, Q and R are collinear points

M(4, -5), N(1, 1), S(-2, 7)

$$MN = \sqrt{(4-1)^2 + (-5-1)^2} = \sqrt{9+36} = 3\sqrt{5} \text{ units}$$

$$NS = \sqrt{(1+2)^2 + (1-7)^2} = \sqrt{9+36} = 3\sqrt{5} \text{ units}$$

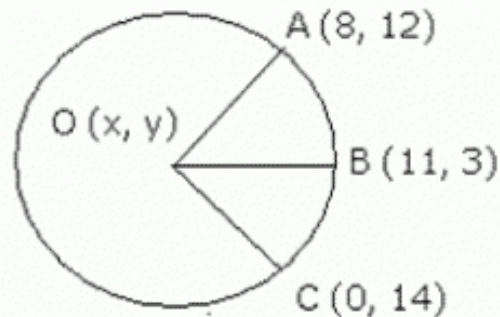
$$MS = \sqrt{(4+2)^2 + (-5-7)^2} = \sqrt{36+144} = 6\sqrt{5} \text{ units}$$

$$MN + NS = 3\sqrt{5} + 3\sqrt{5} = 6\sqrt{5} = MS$$

$$\therefore MN + NS = MS$$

\therefore M, N and S are collinear points.

Answer 16.



Let $O(x, y)$ be the centre of the circle.

$OA = OB$ (radii of the same circle)

$$\Rightarrow OA^2 = OB^2$$

$$(x-8)^2 + (y-12)^2 = (x-11)^2 + (y-3)^2$$

$$\Rightarrow x^2 + 64 - 16x + y^2 + 144 - 24y = x^2 + 121 - 22x + y^2 + 9 - 6y$$

$$\Rightarrow 6x - 18y + 78 = 0$$

$$\Rightarrow x - 3y + 13 = 0 \quad \dots\dots(1)$$

similarly, $OB = OC$

$$\therefore OB^2 = OC^2$$

$$(x-11)^2 + (y-3)^2 = (x-0)^2 + (y-14)^2$$

$$\Rightarrow x^2 + 121 - 22x + y^2 + 9 - 6y = x^2 + y^2 + 196 - 28y$$

$$\Rightarrow -22x + 22y - 66 = 0$$

$$\Rightarrow -x + y - 3 = 0 \quad \dots\dots(2)$$

$$x - 3y + 13 = 0 \quad \dots\dots(1)$$

solving (1) & (2) we get,

$$-2y + 10 = 0$$

$$\Rightarrow y = 5$$

from (1)

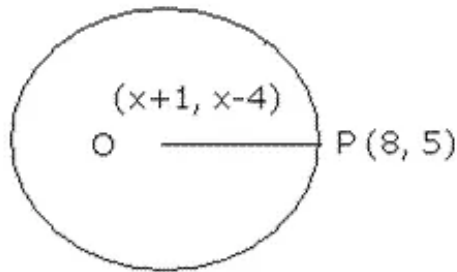
$$x - 15 + 13 = 0$$

$$\Rightarrow x = 2$$

Thus, coordinates of O are $(2, 5)$

$$\text{Radius} = \sqrt{(2-8)^2 + (5-12)^2} = \sqrt{36 + 49} = \sqrt{85} \text{ units}$$

Answer 19.



Given diameter of the circle = 20 units.
 \therefore radius = 10 units

$$OP = 10$$

$$\sqrt{(x+1-8)^2 + (x-4-5)^2} = 10$$

squaring both sides,

$$x^2 + 49 - 14x + x^2 = 81 - 18x = 100$$

$$\Rightarrow 2x^2 - 32x + 30 = 0$$

$$\Rightarrow x^2 - 16x + 15 = 0$$

$$\Rightarrow x^2 - 15x - x + 15 = 0$$

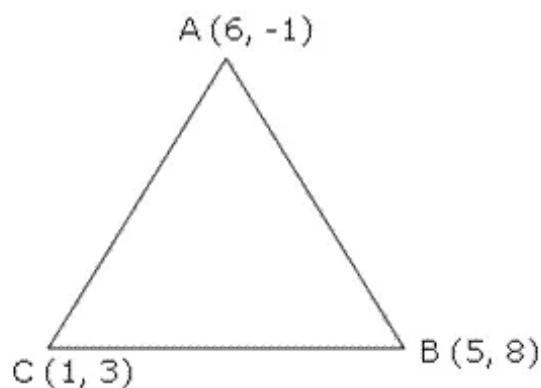
$$\Rightarrow (x-15)(x-1) = 0$$

$$\Rightarrow x = 15 \text{ or } 1$$

Coordinates of O when $x = 15$ are $(16, 11)$

Coordinates of O when $x = 1$ are $(2, -3)$

Answer 23.



$$AB = \sqrt{(6-5)^2 + (-1-8)^2} = \sqrt{1+81} = \sqrt{82} \text{ units}$$

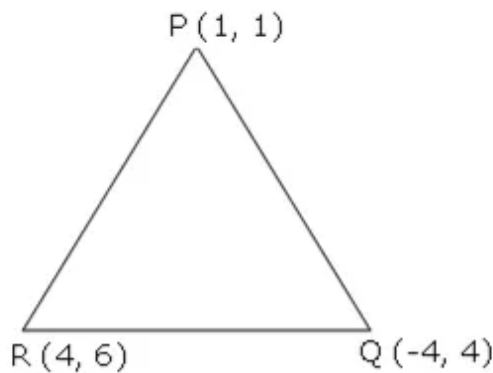
$$BC = \sqrt{(5-1)^2 + (8-3)^2} = \sqrt{16+25} = \sqrt{41} \text{ units}$$

$$AC = \sqrt{(1-6)^2 + (3-1)^2} = \sqrt{25+16} = \sqrt{41} \text{ units}$$

$$\therefore BC = AC,$$

\therefore A, B and C are the vertices of an isosceles triangle.

Answer 24.



$$PQ = \sqrt{(1+4)^2 + (1-4)^2} = \sqrt{25+9} = \sqrt{34} \text{ units}$$

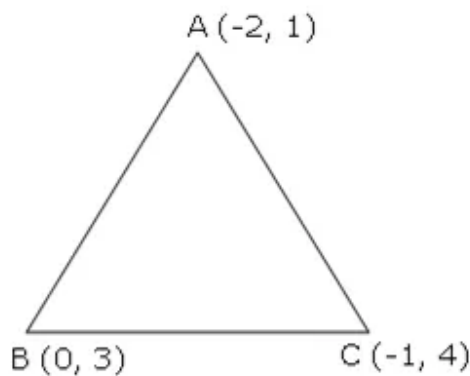
$$QR = \sqrt{(-4-4)^2 + (4-6)^2} = \sqrt{64+4} = \sqrt{68} \text{ units}$$

$$PR = \sqrt{(4-1)^2 + (6-1)^2} = \sqrt{9+25} = \sqrt{34} \text{ units}$$

$$\therefore PQ = QR,$$

\therefore P, Q and R are the vertices of an isosceles triangle

Answer 25.



$$AB = \sqrt{(-2-0)^2 + (1-3)^2} = \sqrt{4+4} = \sqrt{8} \text{ units}$$

$$BC = \sqrt{(1+1)^2 + (3-4)^2} = \sqrt{1+1} = \sqrt{2} \text{ units}$$

$$AC = \sqrt{(-2+1)^2 + (1-4)^2} = \sqrt{1+9} = \sqrt{10} \text{ units}$$

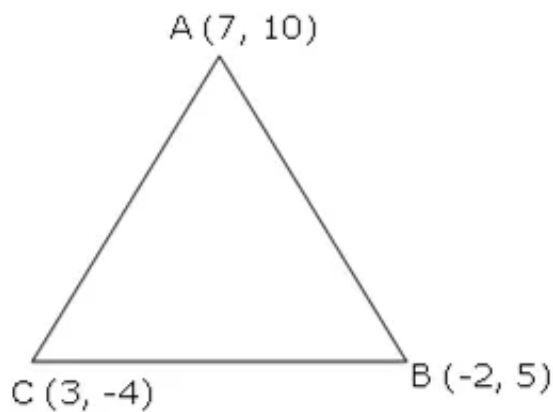
$$AB^2 + BC^2 = 8 + 2 = 10$$

$$AC^2 = 10$$

$$\therefore AB^2 + BC^2 = AC^2$$

\therefore A, B and C are the vertices of a right angled triangle.

Answer 26.



$$AB = \sqrt{(7+2)^2 + (10-5)^2} = \sqrt{81+25} = \sqrt{106} \text{ units}$$

$$BC = \sqrt{(-2-3)^2 + (5+4)^2} = \sqrt{25+81} = \sqrt{106} \text{ units}$$

$$AC = \sqrt{(7-3)^2 + (10+4)^2} = \sqrt{16+196} = \sqrt{212} \text{ units}$$

$$\therefore AB = BC$$

\therefore ABC is an isosceles triangle.

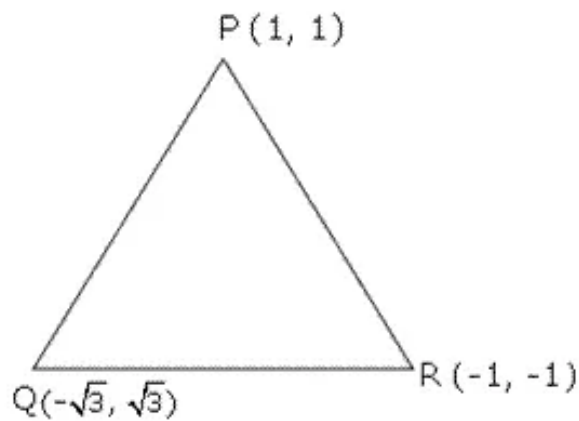
$$AB^2 + BC^2 = 106 + 106 = 212$$

$$AC^2 = 212$$

$$\therefore AB^2 + BC^2 = AC^2$$

\therefore ABC is also a right angled triangle.

Answer 27.



$$PQ = \sqrt{(1 + \sqrt{3})^2 + (1 - \sqrt{3})^2} = \sqrt{4 + 4} = \sqrt{8} \text{ units}$$

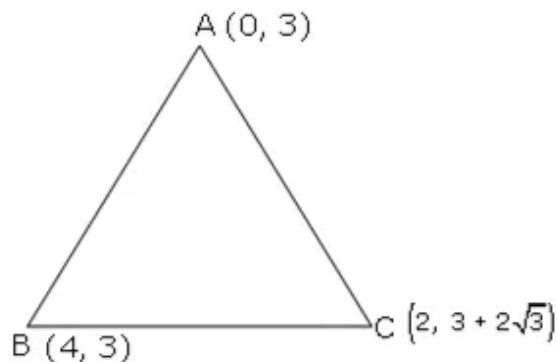
$$QR = \sqrt{(-\sqrt{3} + 1)^2 + (\sqrt{3} + 1)^2} = \sqrt{4 + 4} = \sqrt{8} \text{ units}$$

$$PR = \sqrt{(-1 - 1)^2 + (-1 - 1)^2} = \sqrt{4 + 4} = \sqrt{8} \text{ units}$$

$$\therefore PQ = QR = PR$$

\therefore PQR is an equilateral triangle

Answer 28.



$$AB = \sqrt{(0 - 4)^2 + (3 - 3)^2} = 4 \text{ units}$$

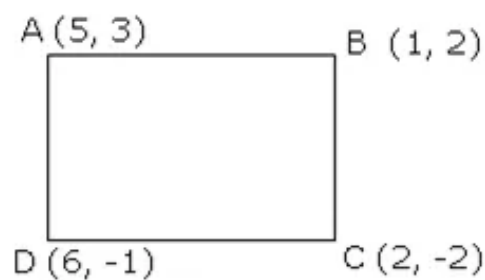
$$BC = \sqrt{(4 - 2)^2 + (3 - 3 - 2\sqrt{3})^2} = \sqrt{4 + 12} = 4 \text{ units}$$

$$AC = \sqrt{(2 - 0)^2 + (3 + 2\sqrt{3} - 3)^2} = \sqrt{4 + 12} = 4 \text{ units}$$

$$\therefore AB = BC = AC$$

\therefore ABC is an equilateral triangle.

Answer 29.



$$AB = \sqrt{(5-1)^2 + (3-2)^2} = \sqrt{16+1} = \sqrt{17} \text{ units}$$

$$BC = \sqrt{(1-2)^2 + (2+2)^2} = \sqrt{1+16} = \sqrt{17} \text{ units}$$

$$CD = \sqrt{(6-2)^2 + (-1+2)^2} = \sqrt{16+1} = \sqrt{17} \text{ units}$$

$$DA = \sqrt{(6-5)^2 + (-1-3)^2} = \sqrt{1+16} = \sqrt{17} \text{ units}$$

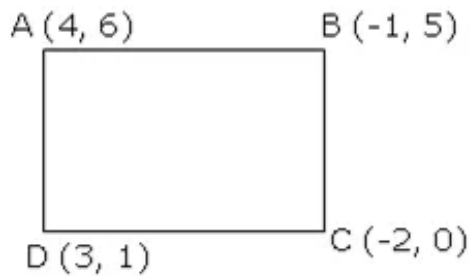
$$AC = \sqrt{(5-2)^2 + (3+2)^2} = \sqrt{9+25} = \sqrt{34} \text{ units}$$

$$BD = \sqrt{(6-1)^2 + (-1-2)^2} = \sqrt{25+9} = \sqrt{34} \text{ units}$$

$\therefore AB = BC = CD = DA$ and $AC = BD$

\therefore ABCD is a square.

Answer 30.



$$AB = \sqrt{(4+1)^2 + (6-5)^2} = \sqrt{25+1} = \sqrt{26} \text{ units}$$

$$BC = \sqrt{(-1+2)^2 + (5-0)^2} = \sqrt{1+25} = \sqrt{26} \text{ units}$$

$$CD = \sqrt{(-2-3)^2 + (0-1)^2} = \sqrt{25+1} = \sqrt{26} \text{ units}$$

$$DA = \sqrt{(3-4)^2 + (1-6)^2} = \sqrt{1+25} = \sqrt{26} \text{ units}$$

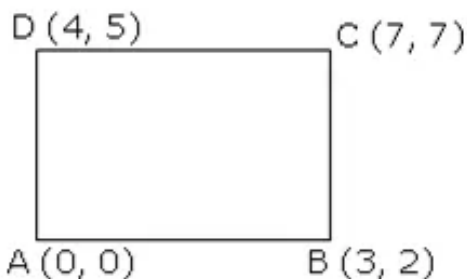
$$AC = \sqrt{(4+2)^2 + (6-0)^2} = \sqrt{36+36} = 36\sqrt{2} \text{ units}$$

$$BD = \sqrt{(-1-3)^2 + (5-1)^2} = \sqrt{36+36} = 16\sqrt{2} \text{ units}$$

$$\therefore AB = BC = CD = DA \text{ and } AC \neq BD$$

\therefore ABCD is a rhombus

Answer 31.



$$AB = \sqrt{(3-0)^2 + (2-0)^2} = \sqrt{9+4} = \sqrt{13} \text{ units}$$

$$BC = \sqrt{(3-7)^2 + (2-7)^2} = \sqrt{16+25} = \sqrt{41} \text{ units}$$

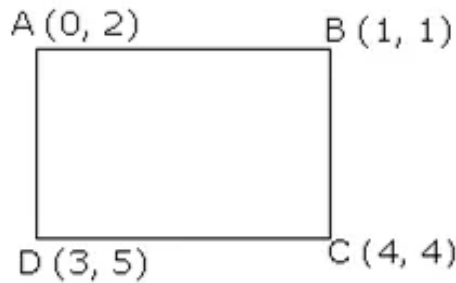
$$CD = \sqrt{(7-4)^2 + (7-5)^2} = \sqrt{9+4} = \sqrt{13} \text{ units}$$

$$DA = \sqrt{(4-0)^2 + (5-0)^2} = \sqrt{16+25} = \sqrt{41} \text{ units}$$

$$\therefore AB = CD \text{ and } BC = DA$$

\therefore ABCD is a parallelogram.

Answer 32.



$$AB = \sqrt{(0-1)^2 + (2-1)^2} = \sqrt{2} \text{ units}$$

$$BC = \sqrt{(1-4)^2 + (1-4)^2} = 3\sqrt{2} \text{ units}$$

$$CD = \sqrt{(4-3)^2 + (4-5)^2} = \sqrt{2} \text{ units}$$

$$DA = \sqrt{(3-0)^2 + (5-2)^2} = 3\sqrt{2} \text{ units}$$

$$AC = \sqrt{(4-0)^2 + (4-2)^2} = \sqrt{20} = 2\sqrt{5} \text{ units}$$

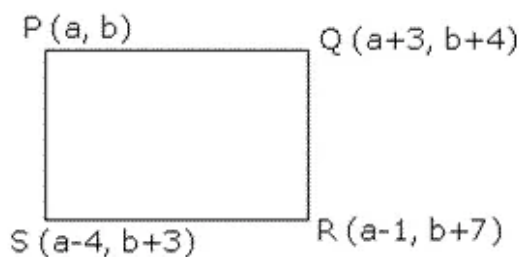
$$BD = \sqrt{(3-1)^2 + (5-1)^2} = \sqrt{20} = 2\sqrt{5} \text{ units}$$

$$\therefore AB = CD \text{ and } BC = DA$$

Also, $AC = BD$

\therefore ABCD is a rectangle.

Answer 33.



$$PQ = \sqrt{(a+3-a)^2 + (b+4-b)^2} = \sqrt{9+16} = 5 \text{ units}$$

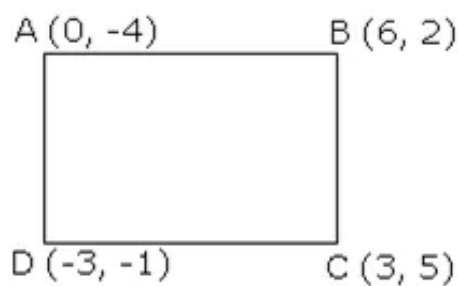
$$QR = \sqrt{(a+3-a+1)^2 + (b-4-b-7)^2} = \sqrt{16+9} = 5 \text{ units}$$

$$RS = \sqrt{(a-1-a+4)^2 + (b+7-b-3)^2} = \sqrt{9+16} = 5 \text{ units}$$

$$SP = \sqrt{(a-4-a)^2 + (b+3-b)^2} = \sqrt{16+9} = 5 \text{ units}$$

Since the opposite sides of quadrilateral PQRS are equal, therefore, it is a parallelogram.

Answer 34.



$$AB = \sqrt{(6-0)^2 + (2+4)^2} = 6\sqrt{2} \text{ units}$$

$$BC = \sqrt{(6-3)^2 + (2-5)^2} = 3\sqrt{2} \text{ units}$$

$$CD = \sqrt{(3+3)^2 + (5+1)^2} = 6\sqrt{2} \text{ units}$$

$$DA = \sqrt{(-3-0)^2 + (-1+4)^2} = 3\sqrt{2} \text{ units}$$

$$AC = \sqrt{(3-0)^2 + (5+4)^2} = 3\sqrt{10} \text{ units}$$

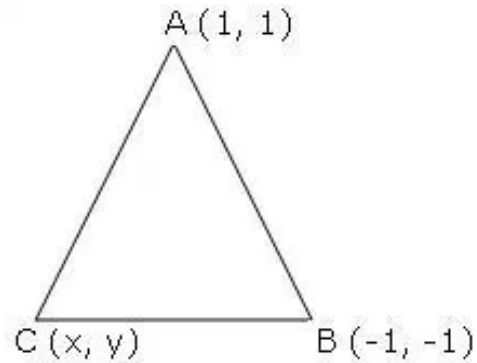
$$BD = \sqrt{(6+3)^2 + (2+1)^2} = 3\sqrt{10} \text{ units}$$

$\therefore AB=CD$ and $BC=DA$,

Also $AC=BD$

\therefore ABCD is a rectangle.

Answer 37.



ABC is an equilateral triangle.

$$\therefore AC = BC \quad \text{and} \quad AB = BC$$

$$\Rightarrow AC^2 = BC^2 \quad \text{and} \quad AB^2 = BC^2$$

$$(x-1)^2 + (y-1)^2 = (x+1)^2 + (y+1)^2$$

$$\Rightarrow x^2 + 1 - 2x + y^2 + 1 - 2y = x^2 + 1 + 2x + y^2 + 1 + 2y$$

$$\Rightarrow -4x - 4y = 0$$

$$\Rightarrow -4x = 4y$$

$$\Rightarrow x = -y \quad \dots\dots(1)$$

$$(1+1)^2 + (1+1)^2 = (x+1)^2 + (y+1)^2$$

$$\Rightarrow 8 = x^2 + 1 + 2x + y^2 + 1 + 2y$$

$$\Rightarrow 8 = y^2 + 1 - 2y + y^2 + 1 + 2y$$

$$\Rightarrow 2y^2 - 6 = 0$$

$$\Rightarrow y^2 = 3$$

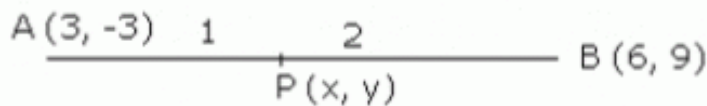
$$\Rightarrow y = \pm \sqrt{3}$$

from(1)

$$\therefore x = \mp \sqrt{3}$$

Ex 12.2

Answer 1.



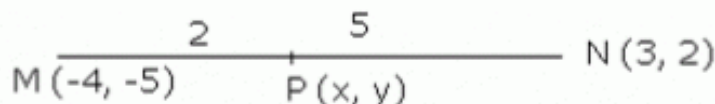
Let the point P divides the line segment AB in the ratio 1:2.

∴ coordinates of P are

$$x = \frac{1 \times 6 + 2 \times 3}{1 + 2} = 4$$

$$y = \frac{1 \times 9 + 2 \times -3}{1 + 2} = 1$$

(b)



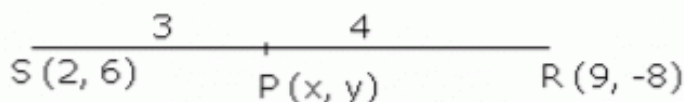
Let the point P divides the line segment MN in the ratio 2:5.

∴ coordinates of P are

$$x = \frac{2 \times 3 + 5 \times -4}{2 + 5} = \frac{-14}{7} = -2$$

$$y = \frac{2 \times 2 + 5 \times -5}{2 + 5} = -3$$

(c)



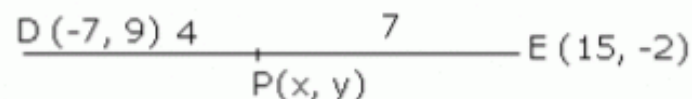
Let the point P divides the line segment SR in the ratio 3:4.

∴ coordinates of P are

$$x = \frac{3 \times 9 + 4 \times 2}{3 + 4} = 5$$

$$y = \frac{3 \times -8 + 4 \times 6}{3 + 4} = 0$$

(d)



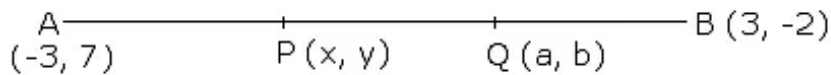
Let the point P divides DE in the ratio 4:7.

∴ coordinates of P are

$$x = \frac{4 \times 15 + 7 \times -7}{4 + 7} = 1$$

$$y = \frac{4 \times -2 + 7 \times 9}{4 + 7} = 5$$

Answer 2.



Let P (x, y) and Q (a, b) be the point of trisection of the line segment AB.

$$AP: PB = 1:2$$

Coordinates of P are

$$x = \frac{1 \times 3 + 2 \times -3}{1 + 2} = -1$$

$$y = \frac{1 \times -2 + 2 \times 7}{1 + 2} = 4$$

$$P(-1, 4)$$

$$AQ:QB = 2:1$$

coordinates of Q are,

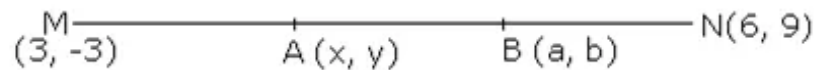
$$a = \frac{2 \times 3 + 1 \times -3}{2 + 1} = 1$$

$$b = \frac{2 \times -2 + 1 \times 7}{2 + 1} = 1$$

$$Q(1, 1)$$

\therefore The points of trisection are $(-1, 4)$ and $(1, 1)$.

Answer 3.



Let A (x, y) and B (a, b) be the points of trisection of line segment MN.

$$MA : AN = 1 : 2$$

\therefore coordinates of A are,

$$x = \frac{1 \times 6 + 2 \times 3}{1 + 2} = 4$$

$$y = \frac{1 \times 9 + 2 \times -3}{1 + 2} = 1$$

$$A(4, 1)$$

$$\text{Also, } MB : BN = 2 : 1$$

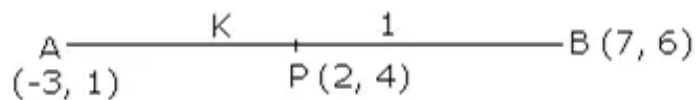
coordinates of B are,

$$a = \frac{2 \times 6 + 1 \times 3}{2 + 1} = 5$$

$$b = \frac{2 \times 9 + 1 \times -3}{2 + 1} = 5$$

$$B(5, 5)$$

points of trisection are $(4, 1)$ and $(5, 5)$.

Answer 4.

Let the point P divides AB in the ratio $k:1$.

Coordinates of P are,

$$x = \frac{7k - 3}{k + 1}$$

$$y = \frac{6k + 1}{k + 1}$$

But given, $P(x, y) = P(2, 4)$

$$\therefore 2 = \frac{7k - 3}{k + 1}$$

$$\Rightarrow 2k + 2 = 7k - 3$$

$$\Rightarrow 5 = 5k$$

$$\Rightarrow k = 1$$

$$k : 1 = 1 : 1$$

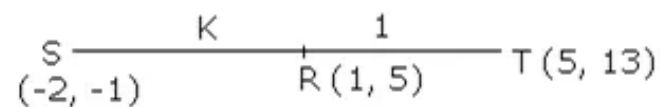
$$\text{or } 4 = \frac{6k + 1}{k + 1}$$

$$4k + 4 = 6k + 1$$

$$\Rightarrow 3 = 2k$$

$$\Rightarrow k = \frac{3}{2}$$

$$k : 1 = 3 : 2$$

Answer 5.

Let R divides the line segment ST in the ratio $k:1$.

Coordinates of R

$$R(x, y) = R(1, 5)$$

$$R\left(\left(\frac{5k - 2}{k + 1}, \frac{13k - 1}{k + 1}\right)\right) = R(1, 5)$$

$$\frac{5k - 2}{k + 1} = 1$$

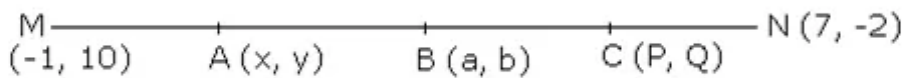
$$5k - 2 = k + 1$$

$$4k = 3$$

$$k = \frac{3}{4}$$

Hence, required ratios $k : 1 = 3 : 4$.

Answer 6.



Given, A (x, y) , B (a, b) and C (p, q) divides the line segment MN in four equal parts. B is the mid point of MN. i.e. MB:BN = 1:1
Coordinates of B are,

$$B(a, b) = B\left(\frac{7-1}{2}, \frac{-2+10}{2}\right) = B(3, 4)$$

A is the mid point of MB i.e. MA : AB = 1 : 1
coordinates of A are.

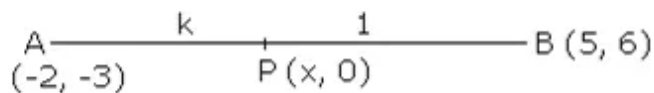
$$A(x, y) = A\left(\frac{3-1}{2}, \frac{4+10}{2}\right) = A(1, 7)$$

C is the mid point of BN i.e. BC : CN = 1 : 1

$$C(p, q) = C\left(\frac{3+7}{2}, \frac{4-2}{2}\right) = C(5, 1)$$

Hence, the coordinates of A, B and C are $(1, 7)$, $(3, 4)$ and $(5, 1)$ respectively.

Answer 7.



Let the point on x-axis be P $(x, 0)$ which divides the line segment AB in the ratio k: 1.

Coordinates of P are

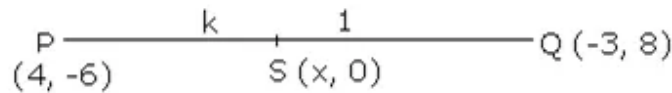
$$x = \frac{5k+2}{k+1}, \quad 0 = \frac{6k-3}{k+1}$$

$$\Rightarrow 0 = 6k - 3$$

$$k = \frac{1}{2}$$

Hence, the required ratio is 1 : 2.

Answer 8.



Given PQ is divided by the line $Y = 0$ i.e. x-axis.

Let S (x, 0) be the point on line $Y = 0$, which divides the line segment PQ in the ratio k: 1.

Coordinates of S are

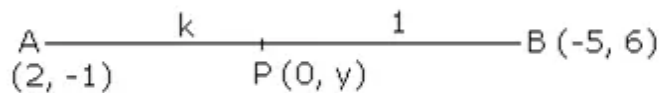
$$x = \frac{-3k + 4}{k + 1}, \quad 0 = \frac{8k - 6}{k + 1}$$

$$\Rightarrow 8k = 6$$

$$\Rightarrow k = \frac{3}{4}$$

Hence, the required ratio is 3 : 4.

Answer 9.



Let the point P (0, y) lies on y-axis which divides the line segment AB in the ratio k: 1.

Coordinates of P are,

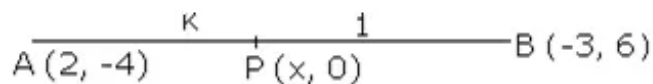
$$0 = \frac{-5k + 2}{k + 1}, \quad y = \frac{6k - 1}{k + 1}$$

$$\Rightarrow 5k = 2$$

$$\Rightarrow k = \frac{2}{5}$$

Hence, the required ratio is 2 : 5.

Answer 10.



Let P (x, 0) be the point on line $y = 0$ i.e. x-axis which divides the line segment AB in the ratio k: 1.

Coordinates of P are

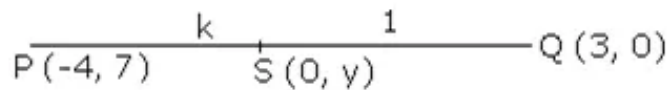
$$x = \frac{-3k + 2}{k + 1}, \quad 0 = \frac{6k - 4}{k + 1}$$

$$\Rightarrow 6k = 4$$

$$\Rightarrow k = \frac{2}{3}$$

Hence the required ratio is 2 : 3.

Answer 11.



Let S (0, y) be the point on line $x = 0$ i.e. y-axis which divides the line segment PQ in the ratio $k: 1$.

Coordinates of S are,

$$0 = \frac{3k - 4}{k + 1}, \quad Y = \frac{0 + 7}{k + 1}$$

$$\Rightarrow 3k = 4$$

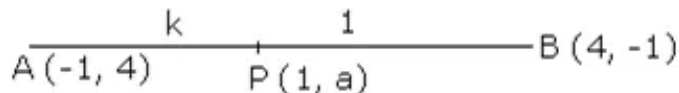
$$k = \frac{4}{3} \quad \dots(1)$$

$$Y = \frac{7}{\frac{4}{3} + 1} \quad (\text{from}(1))$$

$$Y = 3$$

Hence, the required ratio is $4:3$ and the required point is S(0, 3).

Answer 12.



Let the point P (1, a) divides the line segment AB in the ratio $k: 1$.

Coordinates of P are,

$$1 = \frac{4k - 1}{k + 1},$$

$$\Rightarrow k + 1 = 4k - 1$$

$$\Rightarrow 2 = 3k$$

$$\Rightarrow k = \frac{2}{3} \quad \dots(1)$$

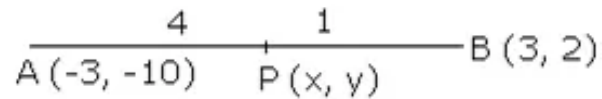
$$\Rightarrow a = \frac{-k + 4}{k + 1}$$

$$\Rightarrow a = \frac{\frac{-2}{3} + 4}{\frac{2}{3} + 1} \quad (\text{from}(1))$$

$$\Rightarrow a = \frac{10}{5} = 2$$

Hence, the required ratio is $2:3$ and the value of a is 2.

Answer 13.



Given : $AP : PB = 4 : 1$

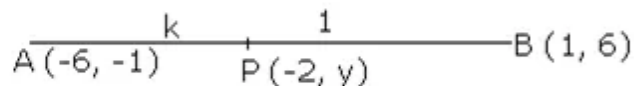
$\therefore AP : AB = 4 : 5$

Coordinates of P are

$$(x, y) = \left(\frac{4 \times 3 - 3}{5}, \frac{4 \times 2 - 10}{5} \right) = \left(\frac{9}{5}, \frac{-2}{5} \right)$$

$$P \left(\frac{9}{5}, \frac{-2}{5} \right)$$

Answer 14.



Let P (-2, y) be the point on line x which divides the line segment AB the ratio k : 1.

Coordinates of P are,

$$-2 = \frac{k - 6}{k + 1},$$

$$\Rightarrow -2k - 2 = k - 6$$

$$\Rightarrow -3k = -4$$

$$\Rightarrow k = \frac{4}{3} \quad \dots (1)$$

$$y = \frac{6k - 1}{k + 1}$$

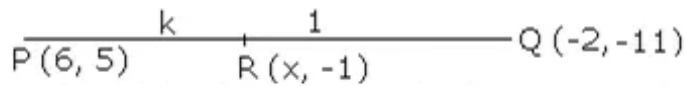
$$\Rightarrow y = \frac{6 \left(\frac{4}{3} \right) - 1}{\frac{4}{3} + 1} \quad (\text{from (1)})$$

$$\Rightarrow y = \frac{24 - 3}{7}$$

$$\Rightarrow y = 3$$

Hence, the required ratio is 4 : 3 and the point of intersection is (-2, 3).

Answer 15.



Let $R(x, -1)$ be the point on the line $y = -1$ which divides the line segment PQ in the ratio $k: 1$.

Coordinates of R are,

$$x = \frac{-2k + 6}{k + 1}, \quad -1 = \frac{-11k + 5}{k + 1}$$

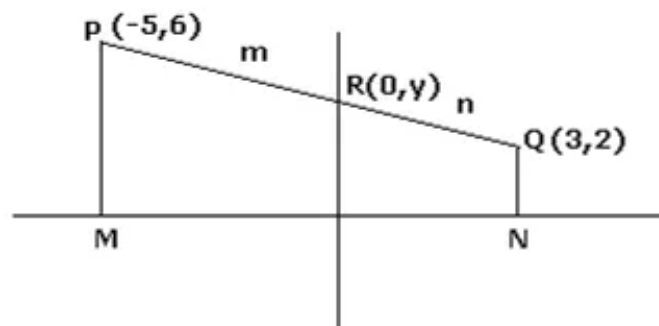
$$x = \frac{-2\left(\frac{3}{5}\right) + 6}{\frac{3}{5} + 1}, \Rightarrow -k - 1 = -11k + 5$$

$$\Rightarrow x = \frac{-6 + 30}{8} \Rightarrow 10k = 6$$

$$x = 3 \quad \Rightarrow k = \frac{3}{5} \dots (1)$$

Hence, the required ratio is $3:5$ and the point of intersection is $(3, -1)$.

Answer 16.



$R(0, y)$ is the point on the y -axis that divides PQ .

Let the ratio in which PQ is divided by R be $m:n$.

Now, $R(0, y), (x_1, y_1) = (-5, 6)$ and $(x_2, y_2) = (3, 2)$ and the ratio is $m:n$.

$$0 = \frac{mx_2 + nx_1}{m + n}$$

$$\Rightarrow 0 = \frac{3m - 5n}{m + n}$$

$$\Rightarrow 0 = 3m - 5n$$

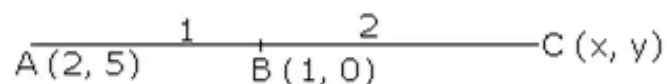
$$\Rightarrow 3m = 5n$$

$$\Rightarrow \frac{m}{n} = \frac{5}{3}$$

$$\Rightarrow m : n = 5 : 3$$

$$\Rightarrow PR : RQ = 5 : 3$$

Answer 17.



Given $AC : AB = 3 : 1$

$\therefore AB : BC = 1 : 2$

Coordinates of B are

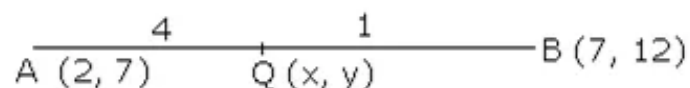
$$1 = \frac{x+4}{3}, \quad 0 = \frac{y+10}{3}$$

$$3 = x + 4, \quad 0 = y + 10$$

$$x = -1, \quad y = -10$$

Hence the coordinates of C are $(-1, -10)$.

Answer 18.



$AQ : BQ = 4 : 1$

Coordinates of Q are

$$Q(x, y) = Q\left(\frac{4 \times 7 + 1 \times 2}{4 + 1}, \frac{4 \times 12 + 1 \times 7}{4 + 1}\right) = Q(6, 11)$$

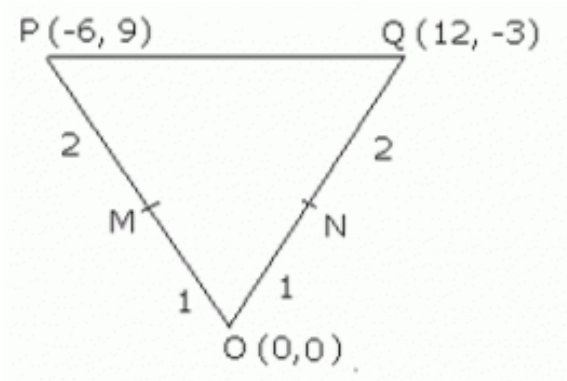
Thus the coordinates of Q are $(6, 11)$.

$$AQ = \sqrt{(2-6)^2 + (7-11)^2} = \sqrt{16+16} = 4\sqrt{2}$$

$$BQ = \sqrt{(7-6)^2 + (12-11)^2} = \sqrt{1+1} = \sqrt{2}$$

$$\Rightarrow AQ = 4BQ$$

Answer 19.



It is given that M divides OP in the ratio 1: 2 and point N divides OQ in the ratio 1: 2.

Using section formula, the coordinates of M are

$$\left(\frac{-6+0}{3}, \frac{9+0}{3} \right) = (-2, 3)$$

Using section formula, the coordinates of N are

$$\left(\frac{12+0}{3}, \frac{-3+0}{3} \right) = (4, -1)$$

Thus, the coordinates of M and N are (-2, 3) and (4, -1) respectively.

Now, using distance formula, we have:

$$PQ = \sqrt{(-6-12)^2 + (9+3)^2} = \sqrt{324+144} = \sqrt{468}$$

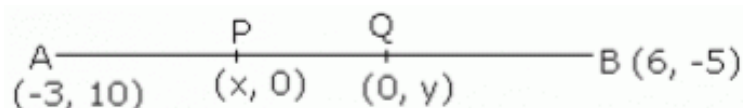
$$MN = \sqrt{(4+2)^2 + (-1-3)^2} = \sqrt{36+36} = \sqrt{52}$$

It can be observed that:

$$PQ = \sqrt{468} = \sqrt{9 \times 52} = 3\sqrt{52} = 3MN$$

Hence, proved.

Answer 22.



Let the coordinates of two points x-axis and y-axis be P (x, 0) and Q (0, y) respectively. Let P divides AB in the ratio k: 1.

Coordinates of P are

$$P(x, 0) = P\left(\frac{6k - 3}{k + 1}, \frac{-5k + 10}{k + 1}\right)$$

$$\Rightarrow 0 = \frac{-5k + 10}{k + 1}$$

$$\Rightarrow 5k = 10$$

$$\Rightarrow k = 2$$

Hence P divides AB in the ratio 2 : 1.

Let Q divides AB in the ratio k_1 : 1.

Coordinates of Q are,

$$Q(0, y) = Q\left(\frac{6k_1 - 3}{k_1 + 1}, \frac{-5k_1 + 10}{k_1 + 1}\right)$$

$$\Rightarrow 0 = \frac{6k_1 - 3}{k_1 + 1}$$

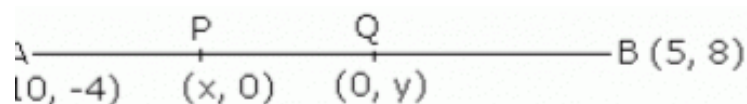
$$\Rightarrow 6k_1 = 3$$

$$\Rightarrow k_1 = \frac{1}{2}$$

Hence Q divides AB in the ratio 1 : 2

Hence proved, P and Q are the points of trisection.

Answer 23.



Let P (x, 0) lies on the line $y = 0$ i.e. x-axis and divides the line segment AB in the ratio $k : 1$.

Coordinates of P are,

$$P(x, 0) = P\left(\frac{5k - 10}{k + 1}, \frac{8k - 4}{k + 1}\right)$$

$$\Rightarrow 0 = \frac{8k - 4}{k + 1}, \quad \frac{5k - 10}{k + 1} = x$$

$$\Rightarrow 8k = 4, \quad \frac{5\left(\frac{1}{2}\right) - 10}{\frac{1}{2} + x} = x \quad (\text{from (1)})$$

$$\Rightarrow k = \frac{1}{2} \dots (1), \quad x = -5$$

Hence P(-5, 0) divides AB in the ratio 1 : 2.

Let Q (0, y) lies on the line $x = 0$ i.e. y-axis and divides the line segment AB in the ratio $k_1 : 1$.

Coordinates of Q are

$$Q(0, y) = Q\left(\frac{5k_1 - 10}{k_1 + 1}, \frac{8k_1 - 4}{k_1 + 1}\right)$$

$$0 = \frac{5k_1 - 10}{k_1 + 1}, \quad y = \frac{8k_1 - 4}{k_1 + 1}$$

$$\Rightarrow 5k_1 = 10, \quad y = \frac{8(2) - 4}{2 + 1} \quad (\text{from (2)})$$

$$\Rightarrow k_1 = 2 \dots (2) \quad y = 4$$

Hence, Q(0, 4) divides in the ratio 2 : 1.

Hence proved P and Q are the points of trisection of AB.

Ex 12.3

Answer 1.

(a)

$$A(4,7) \qquad P(x,y) \qquad B(10,15)$$

Coordinates of P are

$$P(x,y) = P\left(\frac{4+10}{2}, \frac{7+15}{2}\right)$$

$$= P(7,11)$$

(b)

$$P(-3,5) \qquad R(x,y) \qquad Q(9,-9)$$

Coordinates of R are,

$$R(x,y) = R\left(\frac{-3+9}{2}, \frac{5-9}{2}\right)$$

$$= R(3,-2)$$

(c)

$$M(a+b,b-a) \qquad O(x,y) \qquad N(a-b,a+b)$$

Coordinates of O are,

$$O(x,y) = O\left(\frac{a+b+a-b}{2}, \frac{b-a+a+b}{2}\right)$$

$$= O(a,b)$$

(d)

$$A(3a-2b,5a+7b) \qquad C(X,Y) \qquad B(a+4b,a-3b)$$

Coordinates of C are,

$$C(x,y) = C\left(\frac{a+4b+3a-2b}{2}, \frac{a-3b+5a+7b}{2}\right)$$

$$= C(2a+b, 3a+2b)$$

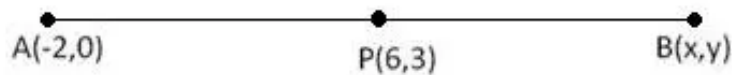
(e)

$$P(a+3,5b) \qquad R(x,y) \qquad Q(3a-1,3b+4)$$

Coordinates of R are,

$$R(x,y) = R\left(\frac{a+3+3a-1}{2}, \frac{5b+3b+4}{2}\right)$$

$$= R(2a+1,4b+2)$$

Answer 2.

Coordinates of P are,

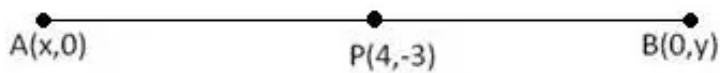
$$P(6,3) = P\left(\frac{-2+x}{2}, \frac{0+y}{2}\right)$$

$$6 = \frac{-2+x}{2}, \quad 3 = \frac{y}{2}$$

$$\Rightarrow 12 = -2 + x, \quad y = 6$$

$$\Rightarrow x = 14$$

Coordinates of B are (14,6).

Answer 3.

Coordinates of B are (14,6)

Let A(x,0) lies on x-axis and B(0,y) lies on y-axis, given AP : PB = 1 : 1

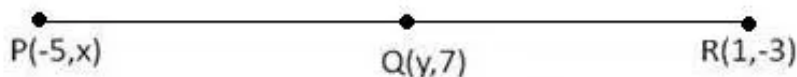
Coordinates of P are,

$$P(4,-3) = P\left(\frac{x+0}{2}, \frac{0+y}{2}\right)$$

$$4 = \frac{x}{2}, \quad -3 = \frac{y}{2}$$

$$x = 8, \quad y = -6$$

Co-ordinates of A are (8,0) and B are (0,-6)

Answer 4.

Given PQ = PR, i.e. PQ : QR = 1 : 1

Coordinates of Q are,

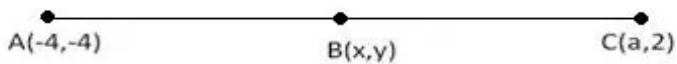
$$Q(y,7) = Q\left(\frac{1-5}{2}, \frac{-3+x}{2}\right)$$

$$y = -2, \quad 7 = \frac{-3+x}{2}$$

$$y = -2, \quad 14 = -3+x$$

$$17 = x$$

The values of x and y are 17 and -2 respectively.

Answer 5.

$$\frac{AB}{AC} = \frac{1}{2}$$

$$\therefore AB : BC = 1 : 1$$

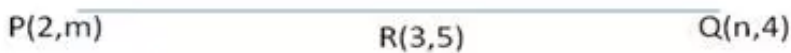
Coordinates of B are,

$$B(-2, b) = B\left(\frac{-4+a}{2}, \frac{-4+2}{2}\right)$$

$$-2 = \frac{-4+a}{2}, \quad b = -1$$

$$-4 = -4 + a, \quad b = -1$$

The values of a and b are 0 and -1 respectively

Answer 6.

$$\text{Given : } PR : RQ = 1 : 1$$

Coordinates of R are,

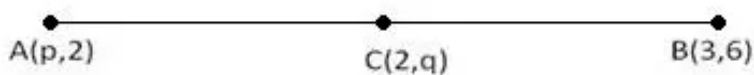
$$R(3, 5) = R\left(\frac{2+n}{2}, \frac{m+4}{2}\right)$$

$$3 = \frac{2+n}{2}, \quad 5 = \frac{m+4}{2}$$

$$6 = 2 + n, \quad 10 = m + 4$$

$$n = 4, \quad m = 6$$

The values of m and n are 6 and 4 respectively.

Answer 7.

$$AC : CB = 1 : 1$$

Coordinates of C are,

$$C(2, q) = C\left(\frac{p+3}{2}, \frac{2+6}{2}\right)$$

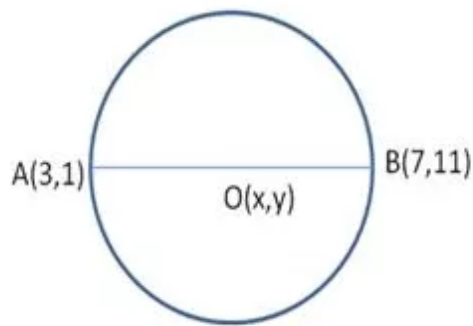
$$2 = \frac{p+3}{2}, \quad q = 4$$

$$4 = p + 3, \quad q = 4$$

$$p = 1, \quad q = 4$$

the values of p and q are 1 and 4 respectively.

Answer 8.



Let O(x,y) be the centre of the circle with diameter AB,

\therefore O is midpoint of AB

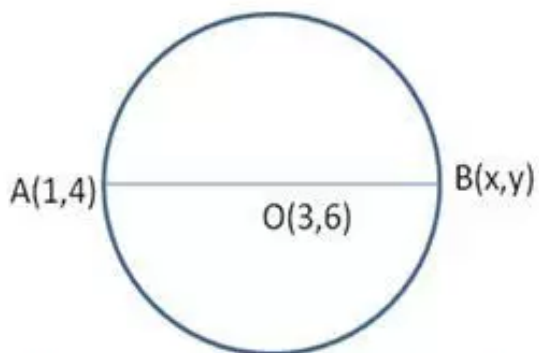
i.e. AO : OB = 1 : 1

Coordinates of O are,

$$O(x,y) = O\left(\frac{3+7}{2}, \frac{1+11}{2}\right) = O(5,6)$$

Thus, the coordinates of centre are (5,6).

Answer 9.



O is the centre of the circle with diameter AB.

\therefore AO : OB = 1 : 1

Coordinates of O are,

$$O(3,6) = O\left(\frac{1+x}{2}, \frac{4+y}{2}\right)$$

$$3 = \frac{1+x}{2}, \quad 6 = \frac{4+y}{2}$$

$$6 = 1 + x \quad 12 = 4 + y$$

$$x = 5, \quad y = 8$$

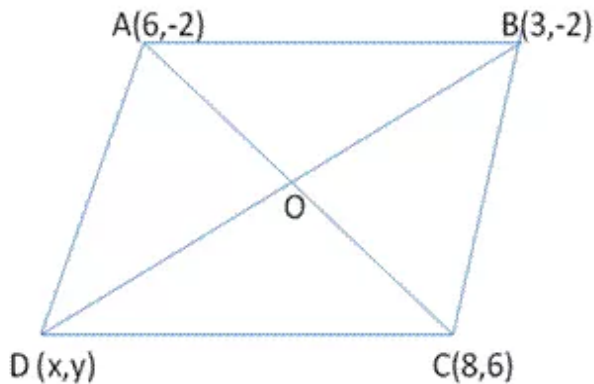
Coordinates of B are (5,8)

$$\text{Length of AB} = \sqrt{(5-1)^2 + (8-4)^2}$$

$$= \sqrt{16 + 16}$$

$$= 4\sqrt{2} \text{ units}$$

Answer 10.



We know that in a parallelogram, diagonals bisect each other.

\therefore midpoint of AC = midpoint of BD

$$O\left(\frac{6+8}{2}, \frac{-2+6}{2}\right) = O\left(\frac{x+3}{2}, \frac{y-2}{2}\right)$$

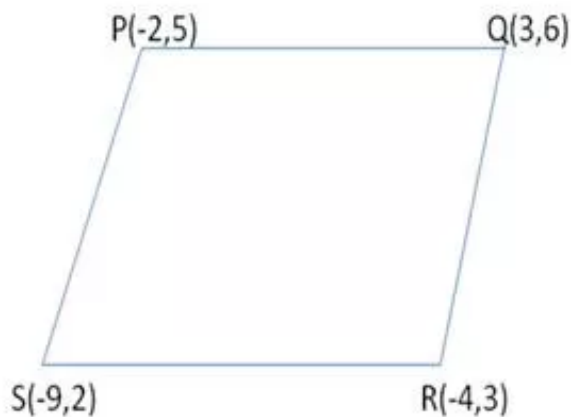
$$\therefore \frac{6+8}{2} = \frac{x+3}{2}, \frac{-2+6}{2} = \frac{y-2}{2}$$

$$14 = x + 3, \quad 4 = y - 2$$

$$x = 11, \quad y = 6$$

the coordinates of the fourth vertex D are (11,6)

Answer 11.



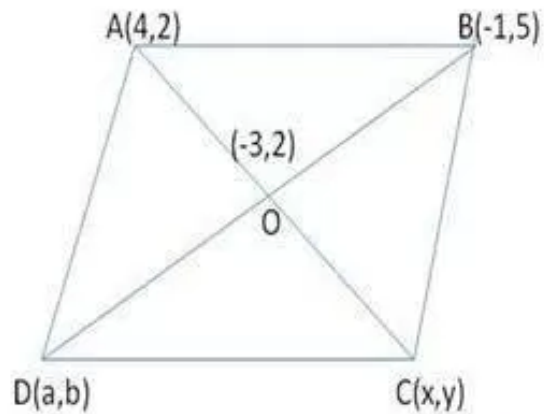
Coordinates of midpoint of PR are $\left(\frac{-2-4}{2}, \frac{5+3}{2}\right)$ i.e. (-3,4)

Coordinates of midpoint of QS are $\left(\frac{-9+3}{2}, \frac{2+6}{2}\right)$ i.e. (-3,4)

The midpoint of PR is same as that of QS, i.e. diagonals PR and QS bisect each other.

Hence, PQRS is a parallelogram.

Answer 12.



Let the coordinates of C and D be (x,y) and (a,b) respectively
Midpoint of AC is O coordinates of O are,

$$O(-3,2) = O\left(\frac{4+x}{2}, \frac{2+y}{2}\right)$$

$$-3 = \frac{4+x}{2}, 2 = \frac{2+y}{2}$$

$$-6 = 4 + x, \quad 4 = 2 + y$$

$$x = -10, \quad y = 2$$

$$C(-10,2)$$

Similarly, coordinates of midpoint of DB, i.e. O are,

$$O(-3,2) = O\left(\frac{a-1}{2}, \frac{b+5}{2}\right)$$

$$-3 = \frac{a-1}{2}, 2 = \frac{b+5}{2}$$

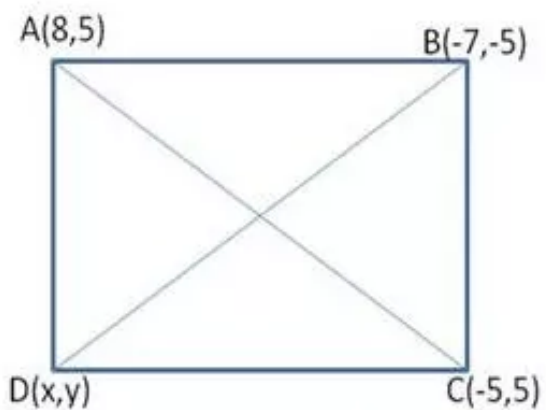
$$-6 = a - 1, 4 = b + 5$$

$$a = -5, b = -1$$

$$D(-5,-1)$$

Thus, the coordinates of the other two vertices are $(-10,2)$ and $(-5,-1)$

Answer 13.



we know that in a parallelogram diagonals bisect each other
 \therefore midpoint of AC = midpoint of BD

$$O\left(\frac{8-5}{2}, \frac{5+5}{2}\right) = O\left(\frac{x-7}{2}, \frac{y-5}{2}\right)$$

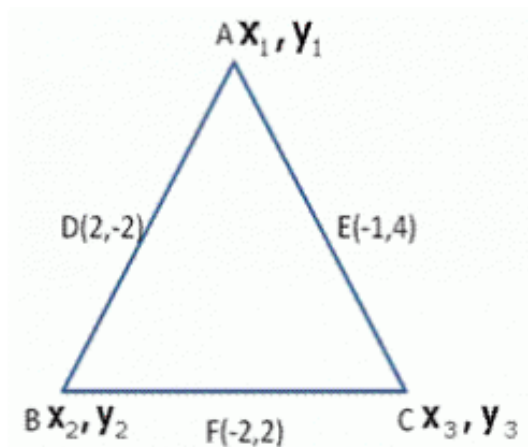
$$\frac{8-5}{2} = \frac{x-7}{2}, \frac{5+5}{2} = \frac{y-5}{2}$$

$$\frac{3}{2} = \frac{x-7}{2}, 10 = y-5$$

$$x = 10, y = 15$$

Coordinates of fourth vertex D are (10,15)

Answer 14.



Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the coordinates of the vertices ΔABC .

Midpoint of AB, i.e. D

$$D(2, -2) = D\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$2 = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} = -2$$

$$x_1 + x_2 = 4 \quad \text{--- (1)} \quad y_1 + y_2 = -4 \quad \text{--- (2)}$$

Similarly,

$$x_1 + x_3 = -2 \quad \text{--- (3)} \quad y_1 + y_3 = 8 \quad \text{--- (4)}$$

$$x_2 + x_3 = -4 \quad \text{--- (5)} \quad y_2 + y_3 = 4 \quad \text{--- (6)}$$

Adding (1), (3) and (5)

$$2(x_1 + x_2 + x_3) = -2$$

$$x_1 + x_2 + x_3 = -1$$

$$4 + x_3 = -1 \text{ [from (1)]}$$

$$x_3 = -5$$

From (3)

$$x_1 - 5 = -2$$

$$x_1 = 3$$

From (5)

$$x_2 - 5 = -4$$

$$x_2 = 1$$

Adding (2), (4) and (6)

$$2(y_1 + y_2 + y_3) = 10$$

$$y_1 + y_2 + y_3 = 5$$

$$-4 + y_3 = 5 \text{ [from (2)]}$$

$$y_3 = 9$$

From (4)

$$y_1 + 9 = 8$$

$$y_1 = -1$$

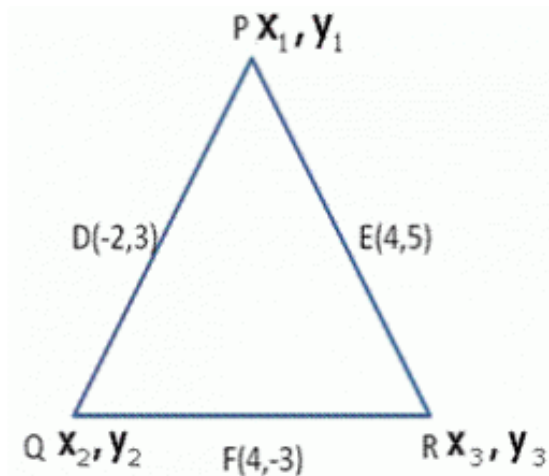
From (6)

$$y_2 + 9 = 4$$

$$y_2 = -5$$

The coordinates of the vertices of ΔABC are $(3, 1)$, $(1, -5)$ and $(-5, 9)$

Answer 15.



Let $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$ be the coordinates of the vertices of ΔPQR .

Midpoint of PQ is D

$$D(-2, 3) = D\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\frac{x_1 + x_2}{2} = -2, \frac{y_1 + y_2}{2} = 3$$

$$x_1 + x_2 = -4 \quad \text{--- (1)}, \quad y_1 + y_2 = 6 \quad \text{--- (2)}$$

similarly,

$$x_2 + x_3 = 8 \quad \text{--- (3)}, \quad y_2 + y_3 = -6 \quad \text{--- (4)}$$

$$x_1 + x_3 = 8 \quad \text{--- (5)}, \quad y_1 + y_3 = 10 \quad \text{--- (6)}$$

Adding (1), (3) and (5)

$$2(x_1 + x_2 + x_3) = 12$$

$$x_1 + x_2 + x_3 = 6$$

$$-4 + x_3 = 6$$

$$x_3 = 10$$

Adding (2), (4) and (6)

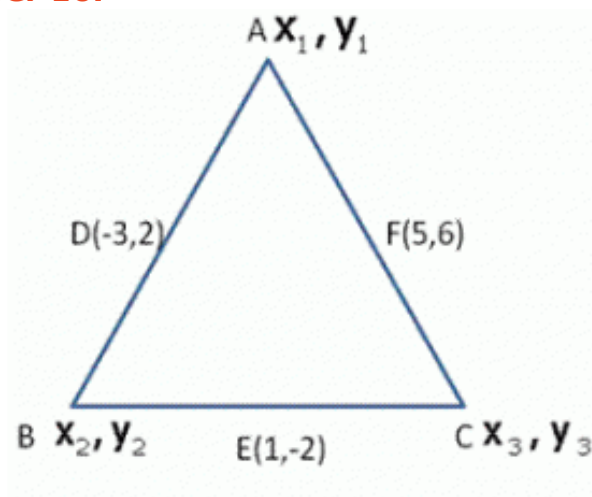
$$2(y_1 + y_2 + y_3) = 10$$

$$y_1 + y_2 + y_3 = 5$$

$$6 + y_3 = 5$$

$$y_3 = -1$$

Answer 16.



let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the coordinates of the vertices of $\triangle ABC$.

D is the midpoint of AB <

$$D(-3, 2) = D\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\frac{x_1 + x_2}{2} = -3, \frac{y_1 + y_2}{2}$$

$$x_1 + x_2 = -6 \text{ --- (1)}$$

Similarly

$$x_2 + x_3 = 2 \text{ --- (3)}$$

$$x_1 + x_3 = 10 \text{ --- (5)}$$

$$y_1 + y_2 = 4 \text{ --- (2)}$$

$$y_2 + y_3 = -4 \text{ --- (4)}$$

$$y_1 + y_3 = 12 \text{ --- (6)}$$

Adding (1), (3) and (5)

$$2(x_1 + x_2 + x_3) = 6$$

$$x_1 + x_2 + x_3 = 3$$

$$-6 + x_3 = 3$$

$$x_3 = 9$$

From (3)

$$x_2 + 9 = 2$$

$$x_2 = -7$$

From (5)

$$x_1 + 9 = 10$$

$$x_1 = 1$$

Adding (2), (4) and (6)

$$2(y_1 + y_2 + y_3) = 12$$

$$y_1 + y_2 + y_3 = 6$$

$$4 + y_3 = 6$$

$$y_3 = 2$$

from(4)

$$y_2 + 2 = -4$$

$$y_2 = -6$$

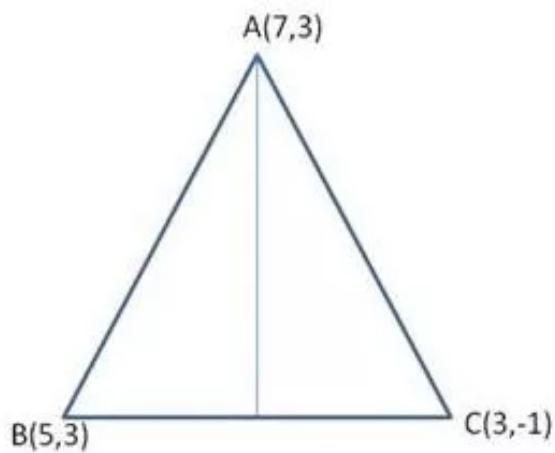
from(6)

$$y_1 + 2 = 12$$

$$y_1 = 10$$

The coordinates of the vertices of $\triangle ABC$ are $(9,2)$, $(1,10)$ and $(-7,-6)$

Answer 17.



we know that the median of triangle bisects the opposite side

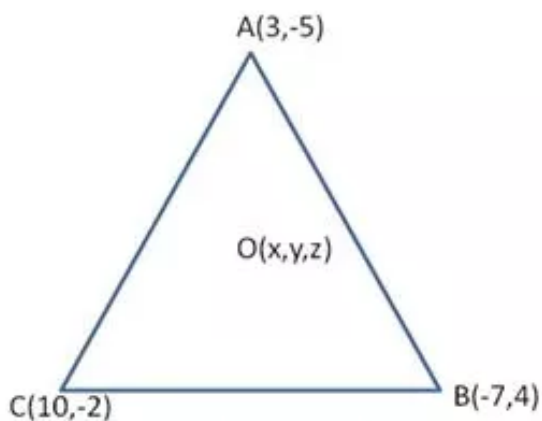
$$\therefore BD : DC = 1 : 1$$

Coordinates of D are,

$$D(x, y) = D\left(\frac{5+3}{2}, \frac{3-1}{2}\right) = D(4, 1)$$

$$\text{Length of median AD} = \sqrt{(7-4)^2 + (-3-1)^2} = \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

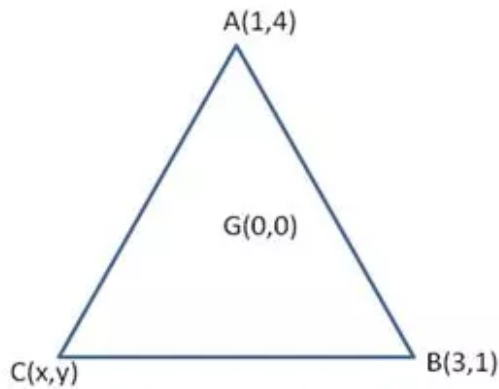
Answer 18.



Let O be the centroid of $\triangle ABC$.

Coordinates of O are

$$\begin{aligned} O(x, y, z) &= O\left(\frac{3+10-7}{3}, \frac{-5+4-2}{3}\right) \\ &= O(2, -1) \end{aligned}$$

Answer 19.

Given the centroid of $\triangle ABC$ is at origin, i.e. $G(0,0)$.

Let the coordinates of third vertex be (x,y) .

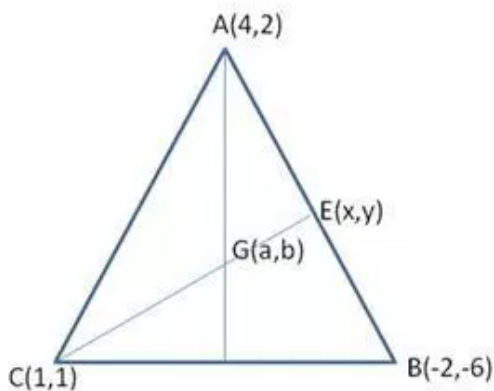
Coordinates of G are,

$$G(0,0) = G\left(\frac{1+3+x}{3}, \frac{4+1+y}{3}\right)$$

$$0 = \frac{4+x}{3}, 0 = \frac{5+y}{3}$$

$$x = -4, y = -5$$

Coordinates of third vertex are $(-4,-5)$

Answer 20.

Let $G(a,b)$ be at centroid of $\triangle ABC$,

Coordinates of G are,

$$G(a,b) = G\left(\frac{4-2+1}{3}, \frac{2-6+1}{3}\right) = G(1,-1)$$

Let CE be the median through C

$$\therefore AE : EB = 1 : 1$$

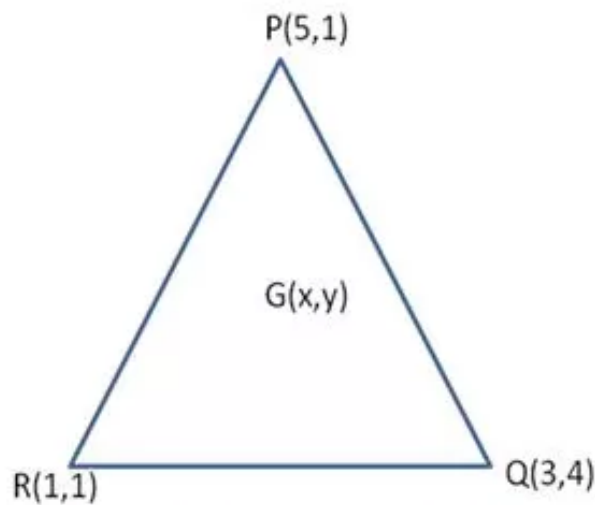
Coordinates of E are

$$E(x,y) = E\left(\frac{4-2}{2}, \frac{2-6}{2}\right) = E(1,-2)$$

Length of median CE

$$\begin{aligned} &= \sqrt{(1-1)^2 + (2-1)^2} \\ &= \sqrt{9} \\ &= 3 \text{ units} \end{aligned}$$

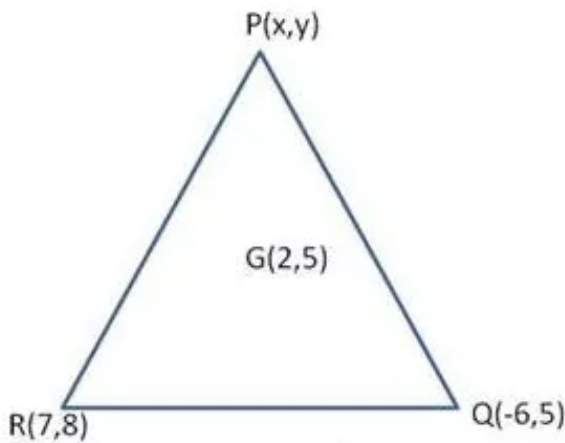
Answer 21.



Let $G(x,y)$ be the centroid of ΔPQR
Coordinates of G are,

$$G(x,y) = G\left(\frac{5+3+1}{3}, \frac{1+4+1}{3}\right) \\ = G(3,2)$$

Answer 22.



Let G be the centroid of ΔPQR whose coordinates are $(2,5)$ and let (x,y) be the coordinates of vertex P .

Coordinates of G are,

$$G(2,5) = G\left(\frac{x-6+7}{3}, \frac{y+5+8}{3}\right)$$

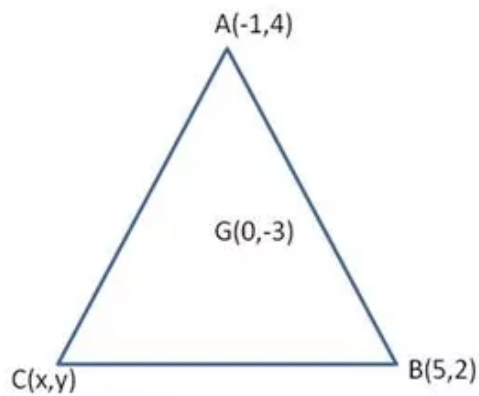
$$2 = \frac{x+1}{3}, 5 = \frac{y+13}{3}$$

$$6 = x + 1, \quad 15 = y + 13$$

$$x = 5, \quad y = 2$$

Coordinates of vertex P are $(5,2)$

Answer 23.



Let G be the centroid of $\triangle ABC$ whose coordinates are (0,-3) and let C(x,y) be the coordinates of third vertex

Coordinates of G are,

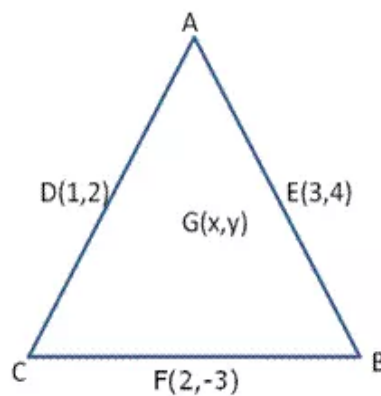
$$G(0, -3) = G\left(\frac{-1+5+x}{3}, \frac{4+2+y}{3}\right)$$

$$0 = \frac{4+x}{3}, -3 = \frac{6+y}{3}$$

$$x = -4, y = -15$$

Coordinates of third vertex are (-4,-15)

Answer 24.



Let ABC be a triangle

The midpoint of whose sides AC, AB and BC are D, E and F respectively.

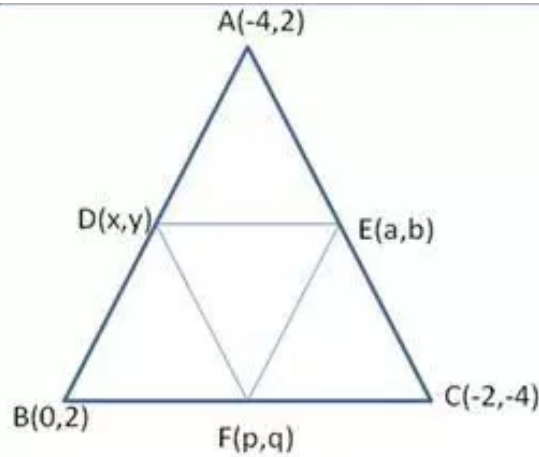
We know that the centroid of $\triangle DEF$. Let G(x,y) be the centroid of $\triangle ABC$ and $\triangle DEF$

Coordinates of centroid G are,

$$G(x, y) = G\left(\frac{1+3+2}{3}, \frac{2+4-3}{3}\right)$$

$$= G(2, 1)$$

Answer 25.



Let D, E and F be the midpoints of the sides AB, AC and BC of $\triangle ABC$ respectively.

$$\therefore AD : DB = BF : FC = AE : EC = 1 : 1$$

Coordinates of D are,

$$D(x, y) = D\left(\frac{0 - 4}{2}, \frac{2 + 2}{2}\right) = D(-2, 2)$$

Similarly,

$$E(a, b) = E\left(\frac{-4 - 2}{2}, \frac{2 - 4}{2}\right) = E(-3, -1)$$

and,

$$F(p, q) = F\left(\frac{0 - 2}{2}, \frac{2 - 4}{2}\right) = F(-1, -1)$$

Coordinates of centroid of $\triangle ABC$ are,

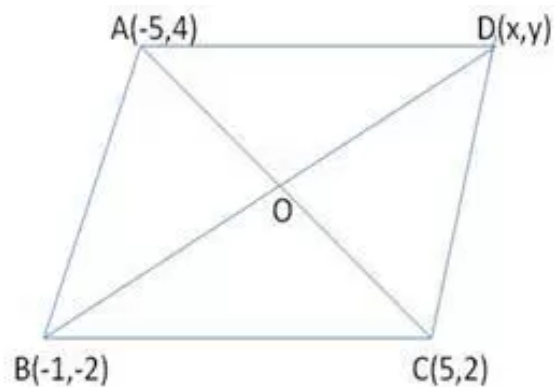
$$= \left(\frac{-4 - 2 + 0}{3}, \frac{2 - 4 + 2}{3}\right) = (-2, 0)$$

Coordinates of centroid of $\triangle DEF$ are,

$$= \left(\frac{-2 - 3 - 1}{3}, \frac{2 - 1 - 1}{3}\right) = (-2, 0)$$

Thus, the centroid of $\triangle ABC$ and $\triangle DEF$ coincides with the centroid of $\triangle DEF$

Answer 26.



$$AB = \sqrt{(-1+5)^2 + (-2-4)^2} = \sqrt{16+36} = \sqrt{52} \text{ units}$$

$$BC = \sqrt{(-1-5)^2 + (-2-2)^2} = \sqrt{36+16} = \sqrt{52} \text{ units}$$

$$AC = \sqrt{(5+5)^2 + (2-4)^2} = \sqrt{100+4} = \sqrt{104}$$

$$AB^2 + BC^2 = 52 + 52 = 104$$

$$AC^2 = 104$$

$$\therefore AB = AC \text{ and } AB^2 + BC^2 = AC^2$$

\therefore ABC is an isosceles right angled triangle.

Let the coordinates of D be (x,y)

If ABCD is a square,

Midpoint of AC = mid point of BD

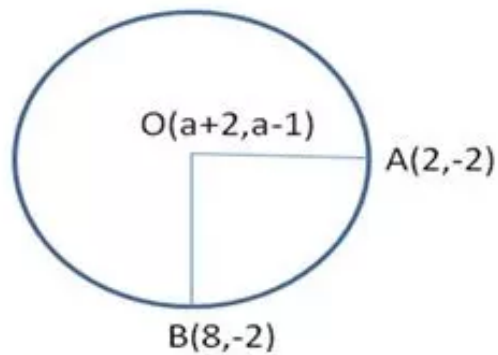
$$O\left(\frac{-5+5}{2}, \frac{4+2}{2}\right) = O\left(\frac{x-1}{2}, \frac{y-2}{2}\right)$$

$$O = \frac{x-1}{2}, 3 = \frac{y-2}{2}$$

$$x = 1, y = 8$$

Coordinates of D are (1,8)

Answer 27.



$OA = OB$ [radii of same circle]

$$\therefore OA^2 = OB^2$$

$$(a+2-2)^2 + (a-1+2)^2 = (a+2-8)^2 + (a-1+2)^2$$

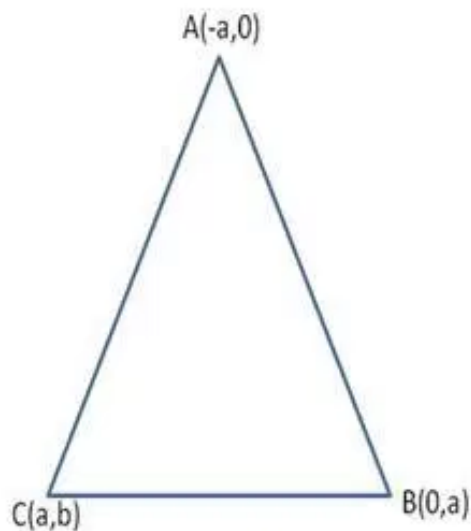
$$a^2 + (a+1)^2 = (a-6)^2 + (a+1)^2$$

$$a^2 = a^2 + 36 - 12a$$

$$12a = 36$$

$$a = 3$$

Answer 28.



Coordinates of G are,

$$G(x, y) = G\left(\frac{-a+0+a}{3}, \frac{0+a+b}{3}\right) = G\left(0, \frac{a+b}{3}\right)$$

$$GA^2 = (0+a)^2 + \left(\frac{a+b}{3} - 0\right)^2$$

$$GA^2 = \frac{9a^2 + a^2 + b^2 + 2ab}{9} = \frac{10a^2 + b^2 + 2ab}{9}$$

$$GB^2 = (0 - 0)^2 + \left(\frac{a+b}{3} - a\right)^2$$

$$GB^2 = \left(\frac{b-2a}{3}\right)^2 = \frac{b^2 + 4a^2 - 4ab}{9}$$

$$GC^2 = (0 - a)^2 + \left(\frac{a+b}{3} - b\right)^2$$

$$GC^2 = a^2 + \left(\frac{a-2b}{3}\right)^2 = \frac{9a^2 + a^2 + 4b^2 - 4ab}{9}$$

$$GA^2 + GB^2 + GC^2 = \frac{10a^2 + b^2 + 2ab + b^2 + 4a^2 - 4ab + 10a^2 + 4b^2 - 4ab}{9}$$

$$= \frac{24a^2 + 6b^2 - 6ab}{9}$$

$$GA^2 + GB^2 + GC^2 = \frac{1}{3}(8a^2 + 2b^2 - 2ab) \dots (1)$$

$$AB^2 = (-a - 0)^2 + (0 - a)^2 = 2a^2$$

$$BC^2 = (0 - a)^2 + (a - b)^2 = a^2 + a^2 + b^2 - 2ab = 2a^2 + b^2 - 2ab$$

$$AC^2 = (-a - a)^2 + (0 - b)^2 = 4a^2 + b^2$$

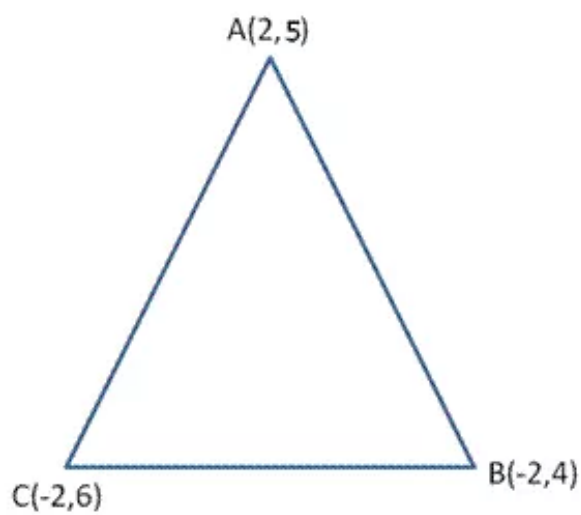
$$AB^2 + BC^2 + AC^2 = 2a^2 + 2a^2 + b^2 - 2ab + 4a^2 + b^2$$

$$AB^2 + BC^2 + AC^2 = 8a^2 + 2b^2 - 2ab \dots (2)$$

from (1) and (2)

$$GA^2 + GB^2 + GC^2 = \frac{1}{3}(AB^2 + BC^2 + AC^2)$$

Answer 29.



$$AB = \sqrt{(2+2)^2 + (5-4)^2} = \sqrt{16+1} = \sqrt{17} \text{ units}$$

$$BC = \sqrt{(-2+2)^2 + (4-6)^2} = \sqrt{0+4} = 2 \text{ units}$$

$$AC = \sqrt{(2+2)^2 + (5-6)^2} = \sqrt{16+1} = \sqrt{17} \text{ units}$$

It can be seen that $AB = AC$.

Hence, the given coordinates are the vertices of an isosceles triangle.