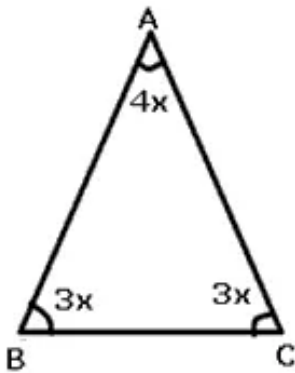


## Chapter 12. Isosceles Triangle

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### Ex 12.1

#### Answer 1.



The equal angles and the non-equal angle are in the ratio 3:4.

Let equal angles be  $3x$  each, therefore non-equal angle is  $4x$ .

Angles of a triangle =  $180^\circ$

$$\Rightarrow 3x + 3x + 4x = 180^\circ$$

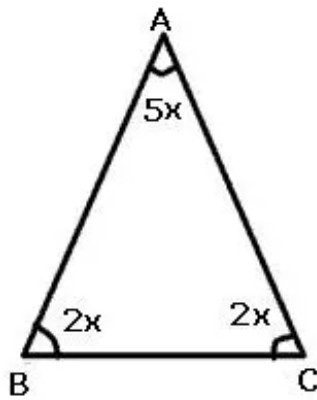
$$\Rightarrow 10x = 180^\circ$$

$$\Rightarrow x = 18^\circ$$

Therefore,  $3x = 54^\circ$  and  $4x = 72^\circ$

Angles =  $54^\circ$ ,  $54^\circ$  and  $72^\circ$

**Answer 2.**



The equal angles and the non-equal angle are in the ratio 2:2:5.

Let equal angles be  $2x$  each, therefore non-equal angle is  $5x$ .

Angles of a triangle =  $180^\circ$

$$\Rightarrow 2x + 2x + 5x = 180^\circ$$

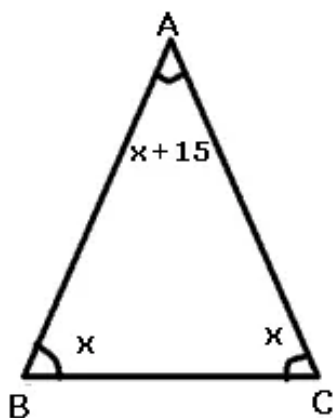
$$\Rightarrow 9x = 180^\circ$$

$$\Rightarrow x = 20^\circ$$

Therefore,  $2x = 40^\circ$  and  $5x = 100^\circ$

Angles =  $40^\circ$ ,  $40^\circ$  and  $100^\circ$

**Answer 3.**



Let equal angles of the isosceles triangle be  $x$  each. Therefore, non-equal angle =  $x+15^\circ$

Angles of a triangle =  $180^\circ$

$$x + x + (x+15^\circ) = 180^\circ$$

$$3x + 15^\circ = 180^\circ$$

$$3x = 165^\circ$$

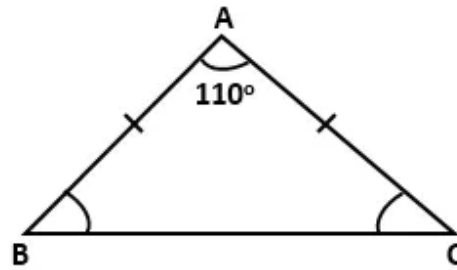
$$x = 55^\circ$$

$$x+15 = 70^\circ$$

Angles are  $55^\circ$ ,  $55^\circ$  and  $70^\circ$



**Answer 4A.**



In  $\triangle ABC$ ,

$$\angle A = 110^\circ$$

$$AB = AC$$

$$\Rightarrow \angle C = \angle B \quad \dots(\text{angles opposite to two equal sides are equal})$$

Now, by angle sum property,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + \angle B + \angle B = 180^\circ$$

$$\Rightarrow 110^\circ + 2\angle B = 180^\circ$$

$$\Rightarrow 2\angle B = 180^\circ - 110^\circ$$

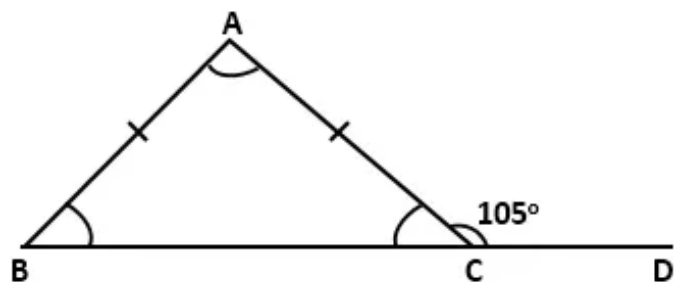
$$\Rightarrow 2\angle B = 70^\circ$$

$$\Rightarrow \angle B = 35^\circ$$

$$\Rightarrow \angle C = 35^\circ$$

Hence,  $\angle B = 35^\circ$  and  $\angle C = 35^\circ$

**Answer 4B.**



In  $\triangle ABC$ ,

$$AB = AC$$

$$\Rightarrow \angle ACB = \angle ABC \quad \dots(1)(\text{angles opposite to two equal sides are equal})$$

Now,  $\angle ACB + \angle ACD = 180^\circ \quad \dots(\text{linear pair})$

$$\Rightarrow \angle ACB = 180^\circ - \angle ACD$$

$$\Rightarrow \angle ACB = 180^\circ - 105^\circ$$

$$\Rightarrow \angle ACB = 75^\circ$$

$$\Rightarrow \angle ABC = 75^\circ \quad \dots[\text{From (1)}]$$

By angle sum property, in  $\triangle ABC$

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ$$

$$\Rightarrow 75^\circ + 75^\circ + \angle BAC = 180^\circ$$

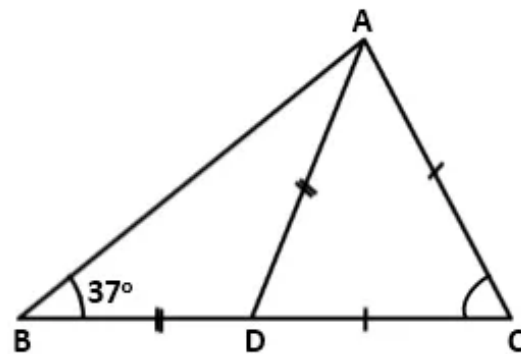
$$\Rightarrow 150^\circ + \angle BAC = 180^\circ$$

$$\Rightarrow \angle BAC = 180^\circ - 150^\circ$$

$$\Rightarrow \angle BAC = 30^\circ$$

Hence, in  $\triangle ABC$ ,  $\angle A = 30^\circ$ ,  $\angle B = 75^\circ$  and  $\angle C = 75^\circ$

#### Answer 4C.



In  $\triangle ABD$ ,

$$AD = BD \quad \dots(\text{given})$$

$$\Rightarrow \angle ABD = \angle BAD \quad \dots(\text{angles opposite to two equal sides are equal})$$

$$\text{Now, } \angle ABD = 37^\circ \quad \dots(\text{given})$$

$$\Rightarrow \angle BAD = 37^\circ$$

By exterior angle property,

$$\angle ADC = \angle ABD + \angle BAD$$

$$\Rightarrow \angle ADC = 37^\circ + 37^\circ = 74^\circ$$

In  $\triangle ADC$ ,

$$AC = DC \quad \dots(\text{given})$$

$$\Rightarrow \angle ADC = \angle DAC \quad \dots(\text{angles opposite to two equal sides are equal})$$

$$\Rightarrow \angle DAC = 74^\circ$$

$$\text{Now, } \angle BAC = \angle BAD + \angle DAC$$

$$\Rightarrow \angle BAC = 37^\circ + 74^\circ = 111^\circ$$

In  $\triangle ABC$ ,

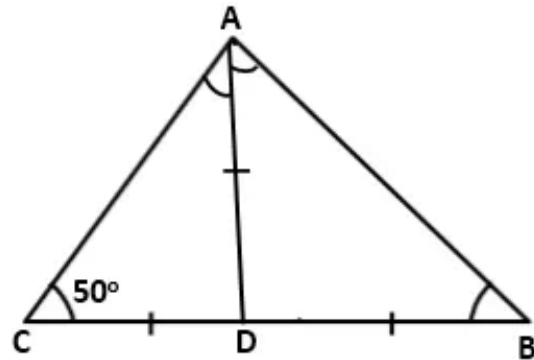
$$\angle BAC + \angle ABC + \angle ACB = 180^\circ$$

$$\Rightarrow 111^\circ + 37^\circ + \angle ACB = 180^\circ$$

$$\Rightarrow \angle ACB = 180^\circ - 111^\circ - 37^\circ = 32^\circ$$

Hence, the interior angles of  $\triangle ABC$  are  $37^\circ$ ,  $111^\circ$  and  $32^\circ$ .

**Answer 4D.**



In  $\triangle ACD$ ,

$AD = CD$  ....(given)

$\Rightarrow \angle ACD = \angle CAD$  ....(angles opposite to two equal sides are equal)

Now,  $\angle ACD = 50^\circ$  ....(given)

$\Rightarrow \angle CAD = 50^\circ$

By exterior angle property,

$$\angle ADB = \angle ACD + \angle CAD = 50^\circ + 50^\circ = 100^\circ$$

In  $\triangle ADB$ ,

$AD = BD$  ....(given)

$\Rightarrow \angle DBA = \angle DAB$  ....(angles opposite to two equal sides are equal)

Also,  $\angle ADB + \angle DBA + \angle DAB = 180^\circ$

$$\Rightarrow 100^\circ + 2\angle DBA = 180^\circ$$

$$\Rightarrow 2\angle DBA = 80^\circ$$

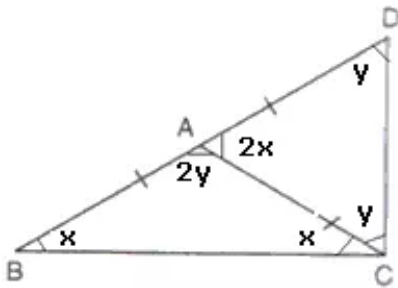
$$\Rightarrow \angle DBA = 40^\circ$$

$$\Rightarrow \angle DAB = 40^\circ$$

$$\angle BAC = \angle DAB + \angle CAD = 40^\circ + 50^\circ = 90^\circ$$

Hence, the interior angles of  $\triangle ABC$  are  $50^\circ, 90^\circ$  and  $40^\circ$ .

**Answer 5.**



Let  $\angle ABC = x$ , therefore  $\angle BCA = x$  since  $AB = AC$   
In  $\triangle ABC$ ,

$$\angle ABC + \angle BCA + \angle BAC = 180^\circ \dots\dots(i)$$

$$\text{But } \angle BAC + \angle DAC = 180^\circ \dots\dots(ii)$$

From (i) and (ii)

$$\angle ABC + \angle BCA + \angle BAC = \angle BAC + \angle DAC$$

$$\angle DAC = \angle ABC + \angle BCA = x + x = 2x$$

Let  $\angle ADC = y$ , therefore  $\angle DCA = y$  since  $AD = AC$

In  $\triangle ADC$ ,

$$\angle ADC + \angle DCA + \angle DAC = 180^\circ \dots\dots(iii)$$

$$\text{But } \angle BAC + \angle DAC = 180^\circ \dots\dots(iv)$$

From (iii) and (iv)

$$\angle ADC + \angle DCA + \angle DAC = \angle BAC + \angle DAC$$

$$\angle BAC = \angle ADC + \angle DCA = y + y = 2y$$

Substituting the value of  $\angle BAC$  and  $\angle DCA$  in (ii)

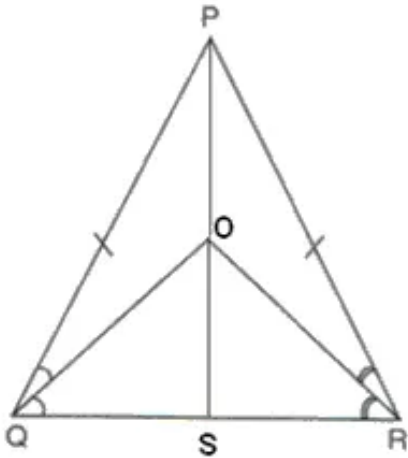
$$2x + 2y = 180^\circ$$

$$x + y = 90^\circ$$

$$\Rightarrow \angle BCA + \angle DCA = 90^\circ$$

$$\Rightarrow \angle BCD \text{ is a right angle.}$$

**Answer 6.**



Join PO and produce to meet QR in S.

In  $\triangle PQS$  and  $\triangle PRS$

$PS = PS$  (common)

$PQ = PR$  (given)

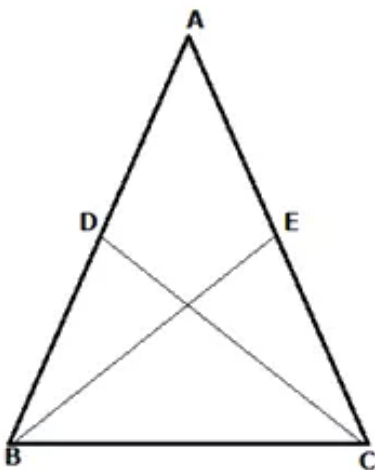
$\angle Q = \angle R$

$\therefore \triangle PQS \cong \triangle PRS$

$\therefore \angle QPS = \angle RPS$

Hence, PO bisects  $\angle P$ .

**Answer 7.**



Let ABC be an isosceles triangle with  $AB=AC$ .

Let D and E be the mid points of AB and AC.

Join BE and CD.

Then BE and CD are the medians of this isosceles triangle.

In  $\triangle ABE$  and  $\triangle ACD$

$$AB = AC \quad (\text{given})$$

$$AD = AE \quad (\text{D and E are mid points of AB and AC})$$

$$\angle A = \angle A \quad (\text{common angle})$$

Therefore,  $\triangle ABE \cong \triangle ACD$  (SAS criteria)

$$\text{Hence, } BE = CD$$

### Answer 8.

In  $\triangle QDP$ ,

$$DP = DQ$$

$$\therefore \angle Q = \angle P$$

$$\angle QDR = \angle Q + \angle P$$

$$2\angle QDC = \angle Q + \angle P \quad (\text{DC bisects angle QDR})$$

$$2\angle QDC = \angle Q + \angle Q = 2\angle Q$$

$$\angle QDC = \angle Q$$

But these are alternate angles.

$$\therefore DC \parallel PQ$$

### Answer 9.

In  $\triangle PQS$ ,

$$PQ = PS$$

$$\therefore \angle PQS = \angle PSQ$$

$$\angle P + \angle PQS + \angle PSQ = 180^\circ$$

$$50^\circ + 2\angle PQS = 180^\circ$$

$$2\angle PQS = 130^\circ$$

$$\angle PQS = 65^\circ = \angle PSQ \dots\dots(i)$$

In  $\triangle SRQ$ ,

$$SR = RQ$$

$$\therefore \angle RQS = \angle RSQ$$

$$\angle R + \angle RQS + \angle RSQ = 180^\circ$$

$$110^\circ + 2\angle RQS = 180^\circ$$

$$2\angle RQS = 70^\circ$$



$$\angle RQS = 35^\circ = \angle RSQ \dots\dots(ii)$$

Adding (i) and (ii)

$$\angle PSQ + \angle RSQ = 65^\circ + 35^\circ$$

$$\angle PSR = 100^\circ$$

**Answer 10.**

In  $\triangle BDC$ ,

$$\angle BDC = 70^\circ$$

$$BD = BC$$

Therefore,  $\angle BDC = \angle BCD$

$$\Rightarrow \angle BCD = 70^\circ$$

$$\text{Now } \angle BCD + \angle BDC + \angle DBC = 180^\circ$$

$$70^\circ + 70^\circ + \angle DBC = 180^\circ$$

$$\angle DBC = 40^\circ$$

$\angle DBC = \angle ABC$  (BC is the angle bisector)

$$\Rightarrow \angle ABC = 40^\circ$$

In  $\triangle ABC$ ,

Since  $AB = AC$ ,  $\angle ABC = \angle ACB \Rightarrow \angle ACB = 40^\circ$

$$\angle ACB + \angle ABC + \angle BAC = 180^\circ$$

$$40^\circ + 40^\circ + \angle BAC = 180^\circ$$

$$\angle BAC = 100^\circ$$

Hence,  $\angle BAC = 100^\circ$  and  $\angle DBC = 40^\circ$

**Answer 11.**

(i) In  $\Delta PQR$ ,

$$PQ = PR \quad (\text{given})$$

$$\therefore \angle R = \angle Q \dots\dots\dots(i)$$

Now in  $\Delta QNT$  and  $\Delta RMT$

$$\angle QNT = \angle RMT \quad (90^\circ)$$

$$\angle Q = \angle R \quad (\text{from (i)})$$

$$QT = TR \quad (\text{given})$$

$$\therefore \Delta QNT \cong \Delta RMT \quad (\text{AAS criteria})$$

$$\therefore TN = TM$$

(ii) Since,  $\Delta QNT \cong \Delta RMT$

$$NQ = MR \dots\dots\dots(ii)$$

$$\text{But } PQ = PR \dots\dots\dots(iii) \quad (\text{given})$$

Subtracting (ii) from (iii)

$$PQ - NQ = PR - MR$$

$$\Rightarrow PN = PM$$

(iii) In  $\Delta PNT$  and  $\Delta PMT$

$$TN = TM \quad (\text{proved})$$

$$PT = PT \quad (\text{common})$$

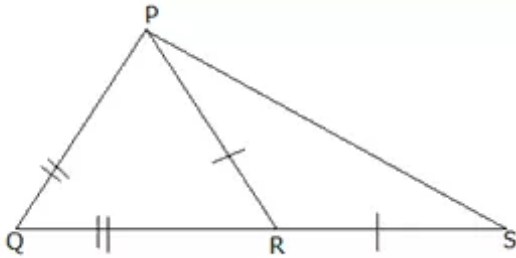
$$\angle PNT = \angle PMT \quad (90^\circ)$$

Therefore,  $\Delta PNT \cong \Delta PMT$

$$\text{Hence, } \angle NPT = \angle MPT$$

Thus,  $PT$  bisects  $\angle P$

**Answer 12.**



In  $\triangle PQR$ ,

$$PQ = QR \quad (\text{given})$$

$$\angle PRQ = \angle QPR \quad \dots\dots(i)$$

In  $\triangle PRS$ ,

$$PR = RS \quad (\text{given})$$

$$\angle PSR = \angle RPS \quad \dots\dots(ii)$$

Adding (i) and (ii)

$$\angle QPR + \angle RPS = \angle PRQ + \angle PSR$$

$$\angle QPS = \angle PRQ + \angle PSR \quad \dots\dots(iii)$$

Now in  $\triangle PRS$ ,

$$\angle PRQ = \angle RPS + \angle PSR$$

$$\angle PRQ = \angle PSR + \angle PSR \quad (\text{from (ii)})$$

$$\angle PRQ = 2\angle PSR \quad \dots\dots(iv)$$

Now,  $\angle QPS = 2\angle PSR + \angle PSR$  (from (iii) and (iv))

$$\angle QPS = 3\angle PSR$$

$$\frac{\angle PSR}{\angle QPS} = \frac{1}{3}$$

$$\Rightarrow \angle PSR : \angle QPS = 1 : 3$$

**Answer 13.**

In  $\triangle KTM$ ,

$$KT = TM \quad (\text{given})$$

$$\text{Therefore, } \angle TKM = \angle TMK \quad \dots(i)$$

$$\text{Now, } \angle KTL = \angle TKM + \angle TMK$$

$$80^\circ = \angle TKM + \angle TKM = 2\angle TKM \quad (\text{from (i)})$$

$$\angle TKM = 40^\circ = \angle TMK = \angle LMK \quad \dots(ii)$$

But  $\angle TKM = \angle TKL$  (KT is the angle bisector)

$$\text{Therefore, } \angle TKL = 40^\circ$$

In  $\triangle KTL$ ,

$$\angle TKL + \angle KTL + \angle KLT = 180^\circ$$

$$40^\circ + 80^\circ + \angle KLT = 180^\circ$$

$$\angle KLT = 60^\circ = \angle KLM$$

$$\angle KLM = 60^\circ \text{ and } \angle LMK = 40^\circ$$

**Answer 14**

In  $\triangle PTQ$  and  $\triangle PSR$

$$PQ = PR \quad (\text{given})$$

$$PT = PS \quad (\text{given})$$

$$\angle TPQ = \angle SPR \quad (\text{vertically opposite angles})$$

Therefore,  $\triangle PTQ \cong \triangle PSR$

Hence,  $TQ = SR$

**Answer 15.**



In  $\triangle ADB$  and  $\triangle ADC$

$AB = AC$  (given)

$AD = AD$  (common)

$\angle BAD = \angle CAD$  (AD bisects  $\angle BAC$ )

Therefore,  $\triangle ADB \cong \triangle ADC$

Hence,  $BD = DC$  and  $\angle BDA = \angle CDA$

But  $\angle BDA + \angle CDA = 180^\circ$

$\Rightarrow \angle BDA = \angle CDA = 90^\circ$

Therefore, AD bisects BC perpendicularly.

**Answer 16.**

In  $\triangle DEC$ ,

$$\angle DEC = \angle ADE + \angle A = 2a \quad (\text{ext. Angle to } \triangle ADE)$$

$$DE = DC$$

$$\Rightarrow \angle DEC = \angle DCE = 2a \quad \dots\dots(ii)$$

In  $\triangle BDC$ , let  $\angle B = b$

$$DC = BC$$

$$\Rightarrow \angle BDC = \angle B = b \quad \dots\dots(iii)$$

In  $\triangle ABC$ ,

$$\angle ADB = \angle ADE + \angle EDC + \angle BDC$$

$$180^\circ = a + \angle EDC + b \quad (\text{from (i) and (iii)})$$

$$\angle EDC = 180^\circ - a - b \quad \dots\dots(iv)$$

Now again in  $\triangle DEC$

$$180^\circ = \angle EDC + \angle DCE + \angle DEC \quad (\text{from (ii)})$$

$$180^\circ = \angle EDC + 2a + 2a$$

$$\angle EDC = 180^\circ - 4a \quad \dots\dots(v)$$

Equating (iv) and (v)

**Answer 17.**

(i) In  $\triangle ADC$ ,

$$AD = AC \quad (\text{given})$$

$$\text{Therefore, } \angle ADC = \angle ACD \dots\dots\dots(i)$$

$$\text{But } \angle ADB + \angle ADC = 180^\circ \dots\dots\dots(ii)$$

$$\text{And } \angle ACD + \angle DCE = 180^\circ \dots\dots\dots(iii)$$

From (i), (ii) and (iii)

$$\angle ADB = \angle DCE$$

(ii) In  $\triangle ABD$  and  $\triangle DCE$

$$BD = CD \quad (\text{D is mid-point of BC})$$

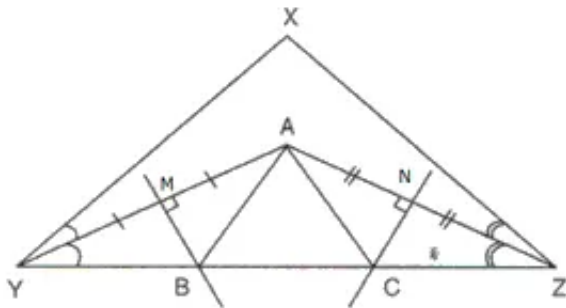
$$\angle ADB = \angle DCE \text{ (proved)}$$

$$AD = CE \quad (\text{since } AD = AC \text{ and } AC = CE)$$

Therefore,  $\triangle ABD \cong \triangle DCE$

Hence,  $AB = CE$ .

**Answer 18.**



Let M and N be the points where AY and AZ are bisected.

In  $\triangle ABM$  and  $\triangle BMY$

$$MY = MA \quad (\text{BM bisects AY})$$

$$BM = BM \quad (\text{common})$$

$$\angle BMY = \angle BMA$$

Therefore,  $\triangle ABM \cong \triangle BMY$

$$\text{Hence, } YB = AB \dots\dots\dots(i)$$

In  $\triangle ACN$  and  $\triangle CNZ$

$$NZ = NA \quad (\text{CN bisects AZ})$$

$$CN = CN \quad (\text{common})$$

$$\angle CAN = \angle CNZ$$

Therefore,  $\triangle ACN \cong \triangle CNZ$

$$\text{Hence, } CZ = AC \dots\dots\dots(ii)$$

$$YZ = YB + BC + CZ$$

Substituting from (i) and (ii)

$$YZ = AB + BC + AC$$

Hence, YZ is equal to the perimeter of  $\triangle ABC$

**Answer 19.**

In  $\triangle PQR$ , let  $\angle PQR = x$

$PQ = PR$

$$\Rightarrow \angle PQR = \angle PRQ = x \quad \dots\dots(i)$$

In  $\triangle RNS$ ,

$$\angle NRS = \angle PRQ = x \quad \dots\dots(\text{vertically opposite angles})$$

$$\angle RNS = 90^\circ \quad (\text{given})$$

$$\angle NSR + \angle RNS + \angle NRS = 180^\circ$$

$$\angle NSR + 90^\circ + x = 180^\circ$$

$$\angle NSR = 90^\circ - x \quad \dots\dots(ii)$$

Now in Quadrilateral PTRS

$$\angle PTS = 90^\circ \quad (\text{given})$$

$$\angle TPR = \angle PQR + \angle PRQ = 2x \quad (\text{exterior angle to triangle PQR})$$

$$\angle PRS = 180^\circ - \angle PRQ = 180^\circ - x \quad (\text{QRS is a st. Line})$$

$$\angle PTS + \angle TPR + \angle PRS + \angle TSR = 360^\circ \quad (\text{angles of a quad.} = 360^\circ)$$

$$90^\circ + 2x + 180^\circ - x + \angle TSR = 360^\circ$$

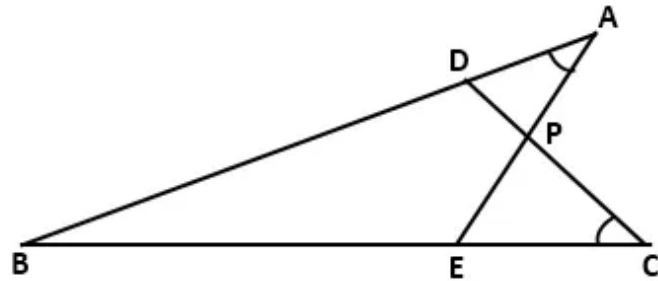
$$\angle TSR = 90^\circ - x \quad \dots\dots(iii)$$

From (ii) and (iii)

$$\angle TSR = \angle NSR$$

Therefore, QS bisects  $\angle TSN$ .

**Answer 20.**



Join DE and AC.

In  $\triangle APD$  and  $\triangle EPC$ ,

$$\angle DAP = \angle ECP \quad \dots (\because \angle BAE = \angle BCD)$$

$$AP = CP \quad \dots (\text{given})$$

$$\angle APD = \angle EPC \quad \dots (\text{vertically opposite angles})$$

$$\therefore \triangle APD \cong \triangle EPC \quad \dots (\text{By ASA Congruence criterion})$$

$$\Rightarrow AD = EC \quad \dots (\text{c.p.c.t})$$

In  $\triangle APC$ ,

$$AP = CP \quad \dots (\text{given})$$

$$\Rightarrow \angle PAC = \angle PCA \quad \dots (\text{angles opposite to two equal sides are equal})$$

Now,  $\angle BAE = \angle BCD$  and  $\angle PAC = \angle PCA$

$$\Rightarrow \angle BAC = \angle BCA$$

$$\Rightarrow BC = BA \quad \dots (\text{sides opposite to two equal angles are equal})$$

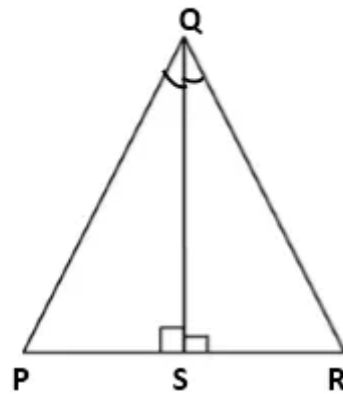
$$\Rightarrow BE + EC = BD + DA$$

$$\Rightarrow BE = BD \quad \dots (\because EC = DA)$$

$$\Rightarrow \angle BDE = \angle BED \quad \dots (\text{angles opposite to two equal sides are equal})$$

$$\Rightarrow \triangle BDE \text{ is an isosceles triangle.}$$

**Answer 21.**



In  $\triangle PQS$  and  $\triangle SQR$ ,

$$QS = QS \quad \dots [\text{Common}]$$

$$\angle QSP = \angle QSR \quad \dots [\text{each} = 90^\circ]$$

$$\angle PQS = \angle RQS \quad \dots [\text{given}]$$

$$\therefore \triangle PQS \cong \triangle SQR \quad \dots [\text{By ASA criterion}]$$

$$\Rightarrow PS = RS$$

$$\Rightarrow x + 1 = y + 2$$

$$\Rightarrow x = y + 1 \quad \dots (i)$$

And  $PQ = SQ$

$$\Rightarrow 3x + 1 = 5y - 2$$

$$\Rightarrow 3(y + 1) + 1 = 5y - 2 \quad \dots[\text{From (i)}]$$

$$\Rightarrow 3y + 3 + 1 = 5y - 2$$

$$\Rightarrow 3y + 4 = 5y - 2$$

$$\Rightarrow 2y = 6$$

$$\Rightarrow y = 3$$

Putting  $y = 3$  in (i),

$$x = y + 1 = 3 + 1 = 4$$

### Answer 22A.

a.

In  $\triangle PQS$ ,

$$PQ = QS \quad \dots(\text{given})$$

$$\Rightarrow \angle QSP = \angle QPS \quad \dots(\text{angles opposite to two equal sides are equal})$$

$$\text{Now, } \angle PQS + \angle QSP + \angle QPS = 180^\circ$$

$$\Rightarrow 60^\circ + 2\angle QSP = 180^\circ$$

$$\Rightarrow 2\angle QSP = 120^\circ$$

$$\Rightarrow \angle QSP = 60^\circ$$

$$\Rightarrow \angle QPS = 60^\circ$$

In  $\triangle PRS$ ,

$$PS = SR \quad \dots(\text{given})$$

$$\Rightarrow \angle PRS = \angle RPS \quad \dots(\text{angles opposite to two equal sides are equal})$$

By exterior angle property,

$$\angle QSP = \angle RPS + \angle PRS$$

$$\Rightarrow 60^\circ = 2\angle RPS$$

$$\Rightarrow \angle RPS = 30^\circ$$

$$\text{Now, } \angle QPR = \angle QPS + \angle RPS = 60^\circ + 30^\circ = 90^\circ$$

b.

In  $\triangle PQS$ ,

$$\angle PQS = 60^\circ, \angle QPS = 60^\circ \text{ and } \angle QSP = 60^\circ$$

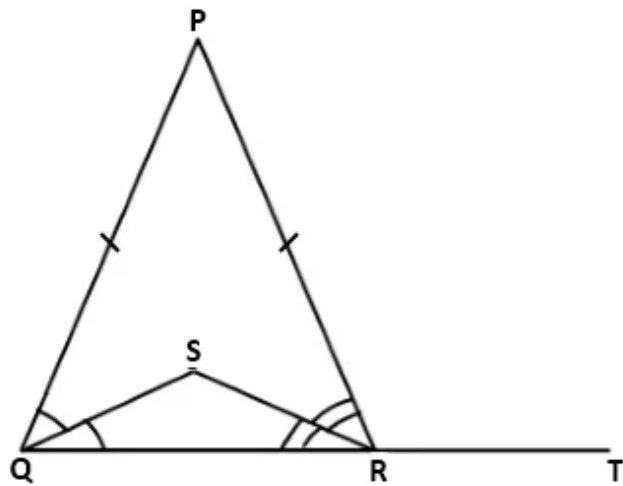
$$\Rightarrow \triangle PQS \text{ is an equilateral triangle.}$$

$$\Rightarrow PQ = QS = PS$$

And,  $PS = SR$

$$\Rightarrow PQ = PS = QS = SR$$

**Answer 23.**



Let  $\angle PQS = \angle SQR = x$  and  $\angle PRS = \angle SRQ = y$

In  $\triangle PQR$ ,

$$\angle QPR + \angle PQR + \angle PRQ = 180^\circ$$

$$\Rightarrow \angle QPR + 2x + 2y = 180^\circ$$

$$\Rightarrow \angle QPR = 180^\circ - 2x - 2y \quad \dots(i)$$

Since  $PQ = PR$ ,

$$\angle PRQ = \angle PQR \quad \dots(\text{angles opposite to two equal sides are equal})$$

$$\Rightarrow 2x = 2y$$

$$\Rightarrow x = y$$

Now,  $\angle PRT = \angle PQR + \angle QPR \quad \dots(\text{by exterior angle property})$

$$\Rightarrow \angle PRT = 2x + 180^\circ - 2x - 2y \quad \dots[\text{From (i)}]$$

$$\Rightarrow \angle PRT = 180^\circ - 2y \quad \dots(ii)$$

In  $\triangle SQR$ ,

$$\angle QSR + \angle SQR + \angle SRQ = 180^\circ$$

$$\Rightarrow \angle QSR + x + y = 180^\circ$$

$$\Rightarrow \angle QSR = 180^\circ - x - y$$

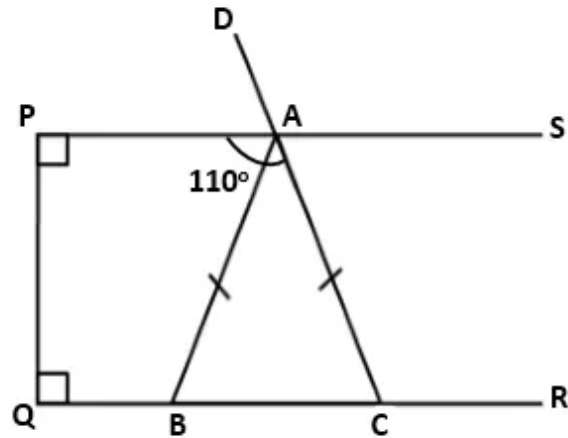
$$\Rightarrow \angle QSR = 180^\circ - y - y \quad \dots[\because x = y \text{ (proved)}]$$

$$\Rightarrow \angle QSR = 180^\circ - 2y \quad \dots(iii)$$

From (ii) and (iii),

$$\angle QSR = \angle PRT$$

**Answer 24.**



Given :  $\angle PAC = 110^\circ$

To find:

Base angles:  $\angle ABC$  and  $\angle ACB$

Vertex angle:  $\angle BAC$

In quadrilateral APQC,

$$\angle APQ + \angle PQC + \angle ACQ + \angle PAC = 360^\circ$$

$$\Rightarrow 90^\circ + 90^\circ + \angle QCA + 110^\circ = 360^\circ$$

$$\Rightarrow \angle ACQ = 360^\circ - 290^\circ$$

$$\Rightarrow \angle ACQ = 70^\circ$$

$$\Rightarrow \angle ACB = 70^\circ \quad \dots(i)$$

In  $\triangle ABC$ ,

$$AB = AC \quad \dots(\text{given})$$

$$\Rightarrow \angle ACB = \angle ABC \quad \dots(\text{angles opposite to two equal sides are equal})$$

$$\Rightarrow \angle ABC = 70^\circ \quad \dots[\text{From (i)}]$$

In  $\triangle ABC$ ,

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ \quad \dots(\text{angle sum property})$$

$$\Rightarrow 70^\circ + 70^\circ + \angle BAC = 180^\circ$$

$$\Rightarrow \angle BAC = 180^\circ - 140^\circ$$

$$\Rightarrow \angle BAC = 40^\circ$$