Chapter 15. Mid-point and Intercept Theorems

Ex 15.1

Answer 1.

In ΔABC,

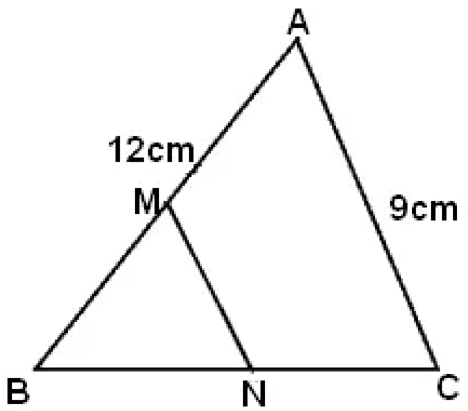
Since D and E are the mid-points of AB and BC respectively

Therefore, by mid-point theorem DE || AC and DE = $\frac{1}{2}$AC

(i) DE = $\frac{1}{2}$AC = $\frac{1}{2}$ x 8.6 cm = 4.3 cm

(ii) $\angle DEB = \angle C = 72°$ (corresponding angles, since DE || AC)

Answer 2.

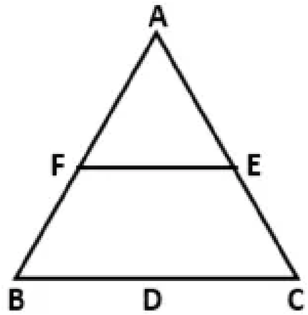


MN || AC and M is mid-point of AB

Therefore, N is mid-point of BC

Hence, MN = $\frac{1}{2}$AC = $\frac{9}{2}$cm = 4.5cm

Answer 3A.



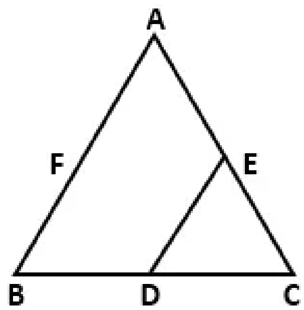
F is the mid-point AB and E is the mid-point of AC.

$\therefore$ FE = $\frac{1}{2}$BC ....(Mid-point Theorem]

= $\frac{1}{2}$ x 14

= 7 cm

Answer 3B.



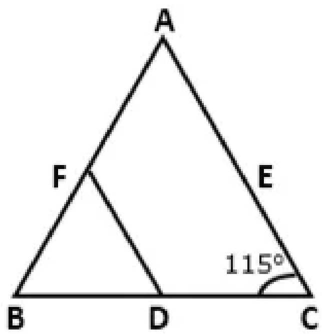
D is the mid-point BC and E is the mid-point of AC.

$\therefore$ DE = $\frac{1}{2}$AB ....(Mid-point Theorem]

= $\frac{1}{2}$ x 8

= 4 cm

Answer 3C.



Here, FD || AC.

$\therefore$ ∠FDB = ∠ACB = 115° ....(Corresponding angles]

Answer 4.

In ΔNSR

$MQ = \frac{1}{2}SR$

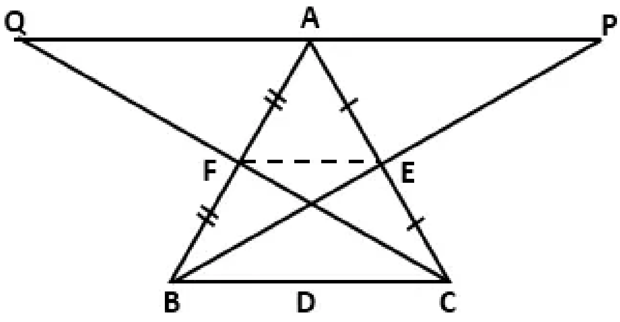
But L is the mid-point of SR and SR = PQ (sides of a parallelogram)

$MQ = \frac{1}{2}PQ$

MQ = PM = LS = LR

Therefore, M is the mid-point of PQ.

Answer 5.



Since BE and CF are medians,
F is the mid-point of AB and E is the mid-point of AC.
Now, the line joining the mid-points of any two sides is parallel and half of the third side, we have

In ΔACQ,

$EF \parallel AQ$ and $EF = \frac{1}{2}AQ$ ....(i)

In ΔABP,

$EF \parallel AP$ and $EF = \frac{1}{2}AP$ ....(ii)

a) From (i) and (ii), we get $AP \parallel AQ$ (both are parallel to EF)
As AP and AQ are parallel and have a common point A, this is possible only if QAP is a straight line.
Hence, proved.

b) From (i) and (ii),

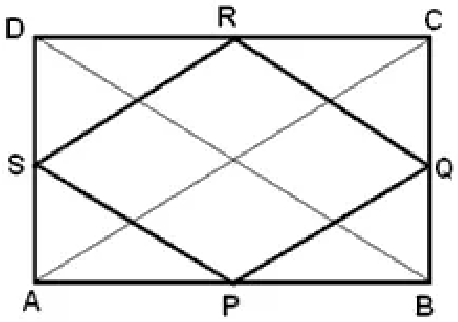
$EF = \frac{1}{2}AQ$ and $EF = \frac{1}{2}AP$

$\Rightarrow \frac{1}{2}AQ = \frac{1}{2}AP$

$\Rightarrow AQ = AP$

$\Rightarrow$ A is the mid-point of QP.

Answer 6.



Join AC and BD.

In ΔABC, P and Q are the mid-points of AB and BC respectively.

$PQ = \frac{1}{2}AC$........(i) and $PQ \parallel AC$

In ΔBDC, R and Q are the mid-points of CD and BC respectively.

$QR = \frac{1}{2}BD$........(ii) and $QR \parallel BD$

But AC = BD (diagonals of a rectangle)

From (i) and (ii)

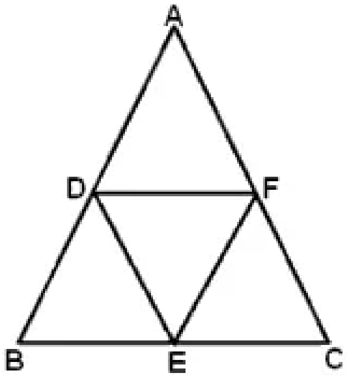
PQ = QR

Similarly, QR = RS, RS = SP and $RS \parallel AC$, $SP \parallel BD$

Hence, PQ = QR = RS = SP

Therefore, PQRS is a rhombus.

Answer 7.



E and F are mid-points of BC and AC

Therefore, EF = $\frac{1}{2}$AB.......(i)

D and F are mid-points of AB and AC

Therefore, DF = $\frac{1}{2}$BC.......(ii)

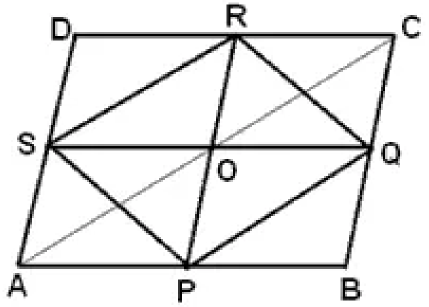
But AB = BC

From (i) and (ii)

EF = DF

Therefore, ΔDEF is an isosceles triangle.

Answer 8.



Join AC.

P and Q are mid-points of AB and BC respectively.

$\therefore$ PQ || AC, $PQ = \frac{1}{2}AC$..........(i)

S and R are mid-points of AD and DC respectively.

$\therefore$ SR || AC, $SR = \frac{1}{2}AC$.........(ii)

From (i) and (ii)

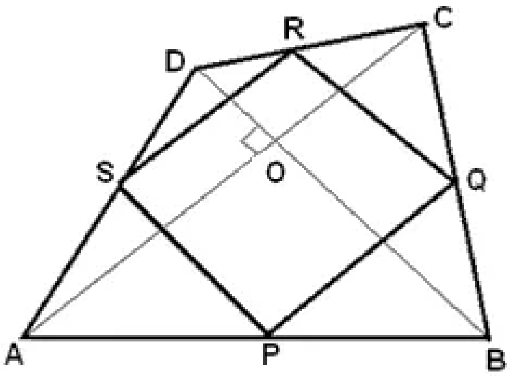
PQ = SR

Therefore, PQRS is a parallelogram.

Since, diagonals of a parallelogram bisect each other

Therefore, PQ and QS bisect each other.

Answer 9.



P and Q are mid-points of AB and BC.

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$AC........(i)

S and R are mid-points of AD and DC.

$\therefore$ SR || AC and SR = $\frac{1}{2}$AC.......(ii)

From (i) and (ii)

PQ || SR and PQ = SR

Therefore, PQRS is a parallelogram.

Further AC and BD intersect at right angles

$\therefore$ SP || BD and BD $\perp$ AC

$\therefore$ SP $\perp$ AC

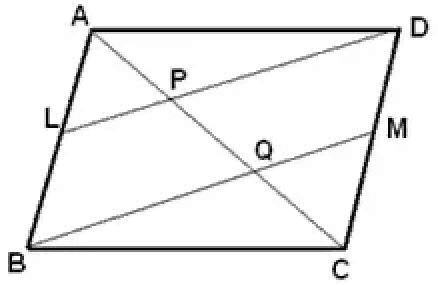
$\Rightarrow$ SP $\perp$ SR

$\Rightarrow \angle RSP = 90^\circ$

$\therefore \angle RSP = \angle SRQ = \angle RQS = \angle SPQ = 90^\circ$

Therefore, PQRS is a rectangle.

Answer 10.



Since L and M are the mid-points of AB and DC respectively.

$BL = \frac{1}{2}AB$ and $DM = \frac{1}{2}DC$.....(i)

But ABCD is a parallelogram

Therefore, AB = CD and AB || DC

⇒BL = DM and BL || DM (from (i))
⇒BLDM is a parallelogram.
⇒DL || BM
⇒LP || BQ ....(ii)

It is known that the segment drawn through the mid-point of one side of a triangle and parallel to the other side bisects the third side.

In ΔABQ, L is the mid-point of AB and MQ || PD

Therefore, P is mid-point of AQ

Hence, AP = PQ ....(iii)

Similarly, in ΔCPD, M is the mid-point of CD and LP || BQ
Therefore, Q is mid-point of CP

Hence, PQ = QC ....(iv)

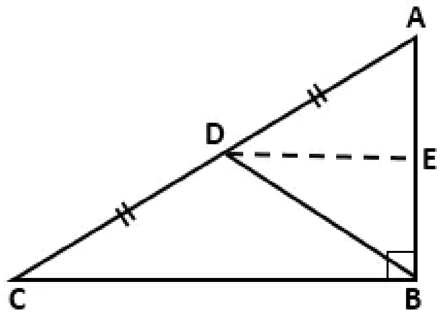
From (iii) and (iv)

AP = PQ = QC

Therefore, P and Q trisect AC

Thus, DL and BM trisect AC.

Answer 11.



Draw line segment DE || CB, which meets AB at point E.

Now, DE || CB and AB is the transversal,

$\therefore \angle AED = \angle ABC$ ....(Corresponding angles)

$\angle ABC = 90^\circ$ ....(given)

$\Rightarrow \angle AED = 90^\circ$

Also, as D is the mid-point of AC and DE || CB,

DE bisects side AB,

i.e. AE = BE ....(i)

In ΔAED and ΔBED,

$\angle AED = \angle BED$ ....(Each 90°)

AE = BE ....[From (i)]

DE = DE ....(Common)

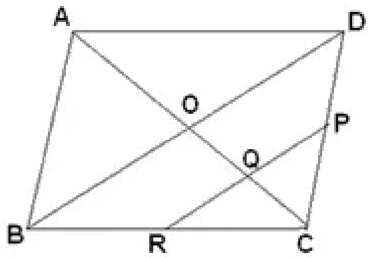
$\therefore \Delta AED \cong \Delta BED$ ....(By SAS Test)

$\Rightarrow AD = BD$ ....(C.P.C.T.C)

$\Rightarrow BD = AC$

$\Rightarrow BD = \frac{1}{2}AC$

Answer 12.



(i) Join B and D. Suppose AC and BD cut at O. Then,

$OC = \frac{1}{2}AC$

Now, $CQ = \frac{1}{4}AC \Rightarrow CQ = \frac{1}{2}OC$

In ΔDCO, P and Q are the mid-points of DC and OC respectively.

$\therefore PQ \parallel DO$

Also, in ΔCOB, Q is the mid-point of OC and PQ || OB

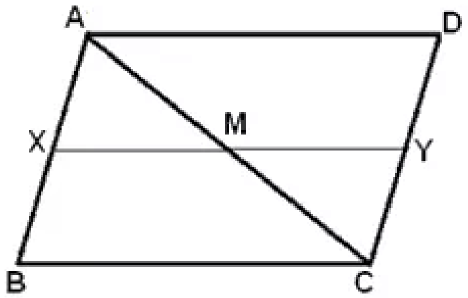
Therefore, R is the mid-point of BC, R being PQ produced.

(ii) In ΔBCD, P and R are the mid-points of DC and BC respectively.

Also PR || BD

Therefore, $PR = \frac{1}{2}BD$

Answer 13.



(i) Join XM and MY.

In ΔAXM and ΔCYM

AM = MC (given)

AX = CY (given)

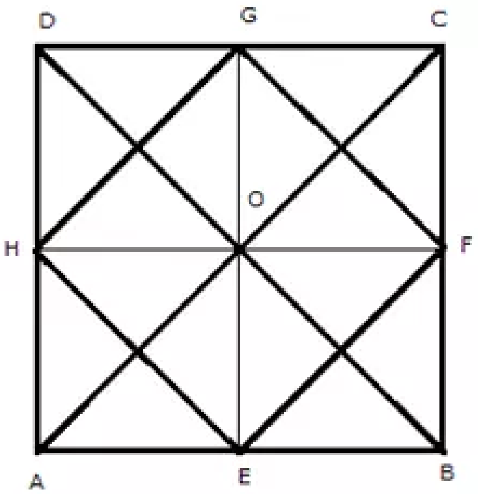
$\angle XAM = \angle YCM$ (alternate angles)

Therefore, $\Delta AXM \cong \Delta CYM$

(ii) $\angle AMX = \angle CMY$ (Vertically opposite angles)

Therefore, XMY is a straight line.

Answer 14.



Join AC and BD

In ΔACD, G and H are the mid-points of DC and AD respectively.

Therefore, $GH \parallel AC$ and $GH = \frac{1}{2}AC$ ....(i)

In ΔABC, E and F are the mid-points of AB and BC respectively.

Therefore, $EF \parallel AC$ and $EF = \frac{1}{2}AC$ ....(ii)

From (i) and (ii)

$EF \parallel GH$ and $EF = GH = \frac{1}{2}AC$ ....(iii)

Similarly it can be proved that-

$EH \parallel GF$ and $EH = GF = \frac{1}{2}BD$ ....(iv)

But AC = BD (diagonals of a square are equal)

Dividing both sides by 2,

$\frac{1}{2}BD = \frac{1}{2}AC$ (iv)

From (iii) and (iv)

EF = GH = EH = GF

Therefore, EFGH is a parallelogram.

Now in ΔGOH and ΔGOF

OH = OF (diagonals of a parallelogram bisect each other)

OG = OG (common)

GH = GF

$\therefore \Delta GOH \cong \Delta GOF$

$\therefore \angle GOH = \angle GOF$

NOW,

$\angle GOH + \angle GOF = 180^\circ$

$\Rightarrow \angle GOH + \angle GOH = 180^\circ$

$\Rightarrow 2\angle GOH = 180^\circ$

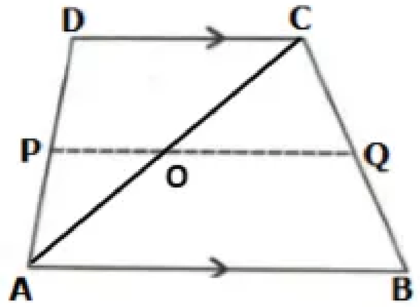
$\Rightarrow \angle GOH = 90^\circ$

Therefore, diagonals of parallelogram EFGH bisect each other and are perpendicular to each other.

Thus, EFGH is a square.

Answer 15A.

Let us draw a diagonal AC which meets PQ at O as shown below:



a) Given AB = 12 cm and DC = 10 cm

In ΔABC,

$OQ = \frac{1}{2}AB$ ....(Mid-point Theorem)

$\Rightarrow OQ = \frac{1}{2} \times 12 = 6$ cm

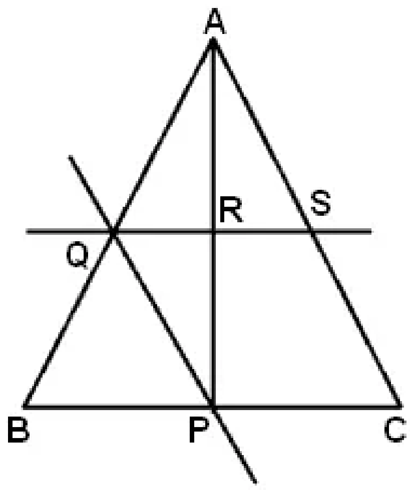
In ΔADC,

$OP = \frac{1}{2}DC$ ....(Mid-point Theorem)

$\Rightarrow OP = \frac{1}{2} \times 10 = 5$ cm

Now, PQ = OP + OQ = 6 + 5 = 11 cm

Answer 16.



(i) In ΔABC,

P is the mid-point of BC and PQ is parallel to AC

Therefore, Q is the mid-point of AB.

In ΔABP,

Q is the mid-point of AB and QR is parallel to BP

Therefore, R is the mid-point of AP.

AR = RP

But AR + RP = AP

$\Rightarrow$ AR + AR = AP

$\Rightarrow$ 2AR = AP or AP = 2AR

(ii) In ΔABC,

Q and S are the mid-points of AB and AC respectively. Also QS is parallel to BC

Therefore, $QS = \frac{1}{2}BC$ ......(i)

Now, AP is the median, hence it bisects BC and QS

Therefore

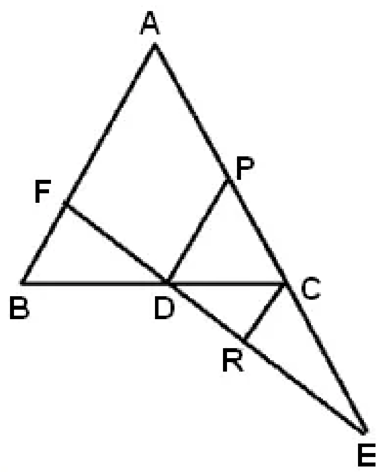
$\frac{1}{2}QS = QR \Rightarrow QS = 2QR$

Substituting in (i)

$\Rightarrow 2QR = \frac{1}{2}BC$

$\Rightarrow BC = 4QR$

Answer 17.



(i) In ΔBDF and ΔDRC,

BD = DC (D is the mid-point of BC)

CR || PD || AB

$\angle$BFD = DRC (alternate angles)

$\angle$BDF = RDC (vertically opposite angles)

Therefore,

$\Delta BDF \cong \Delta DRC$

$\Rightarrow$ DF = DR ......(i)

In ΔABC,

D is the mid-point of BC and DP || AB

Therefore, P is the mid-point of AC.

In ΔDEP,

C is the mid-point of PE and DP || RC|| AB ($CE = \frac{1}{2}AC$ and P is the mid-point of AC)

Therefore, R is the mid-point of DE.

$\Rightarrow DR = RE$ ....(ii)

But $EF = DF + DR + RE$

$EF = DF + DF + DF$

$EF = 3DF$

(ii) In ΔDEP,

C and R are the mid-points of PE and DE respectively.

Also, DP || RC

$\therefore CR = \frac{1}{2}DP$ ....(i)

In ΔABC,

D and P are the mid-points of BC and AC respectively.

Also, DP || AB

$\therefore DP = \frac{1}{2}AB$ ....(ii)

Substituting the value of DP from (ii) in (i)

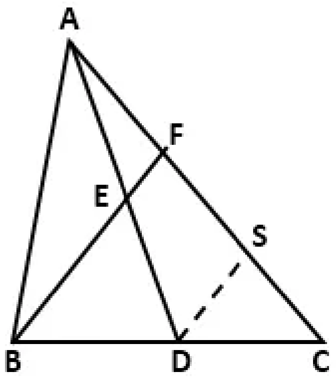
$\Rightarrow CR = \frac{1}{2}(\frac{1}{2}AB)$

$\Rightarrow CR = \frac{1}{4}AB$

$\therefore 4CR = AB$

Answer 18.

Construction: Draw DS || BF, meeting AC at S.



Proof :

In ΔBCF, D is the mid-point of AC and DS || BF.

$\therefore$ S is the mid-point of CF.

$\Rightarrow CS = SF$ ....(i)

In ΔADS, E is the mid-point of AD and EF || DS.

$\therefore$ F is the mid-point of AS.

$\Rightarrow AF = FS$ ....(ii)

From (i) and (ii), we get

$AF = FS = SC$

$\Rightarrow AC = AF + FS + SC$

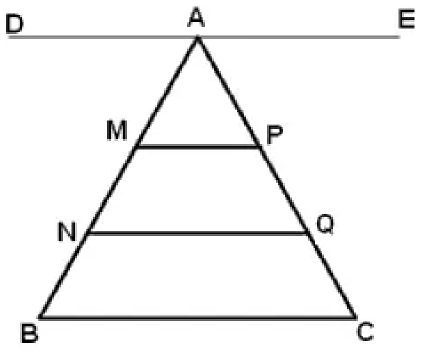
$\Rightarrow AC = AF + AF + AF$

$\Rightarrow AC = 3AF$

$\Rightarrow \frac{AF}{AC} = \frac{1}{3}$

$\Rightarrow AF : AC = 1 : 3$

Answer 19.



Draw DE || BC through A

AM=MN=NB (given)

MP||BC; NQ||BC (given)

DE||BC

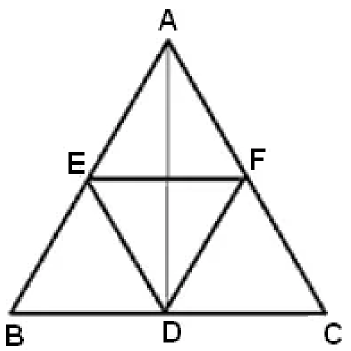
i.e. AM, MN and NB are equal intercepts made on transversal AB.

AC is also a transversal; intercepts made on AC are AP, PQ and QC.

Hence, AP=PQ=QC

Therefore, P and Q divide AC in three equal parts.

Answer 20.



Since the segment joining the mid-points of two sides of a triangle is parallel to third side and is half of it,

Therefore,

$DE \parallel AB, DE = \frac{1}{2}AB$

Also,

$DF \parallel AC, DF = \frac{1}{2}AC$

But AB = AC

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow$ DF = DE ....(i)

$DE = \frac{1}{2}AB \Rightarrow DE = AF$ ......(ii)

From (i), (ii) and (iii)

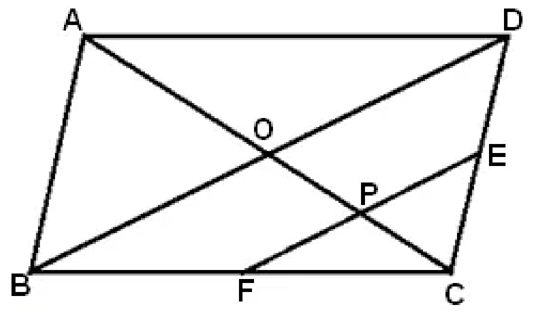
DE = AE = EF = DF

$\Rightarrow$DEAF is a rhombus.

$\Rightarrow$Diagonals AD and EF bisect each other at right angles.

$\Rightarrow$AD perpendicular to EF and AD is bisected by EF.

Answer 21.



(i) Join B and D. Suppose AC and BD cut at O. Then,

$OC = \frac{1}{2}AC$

Now, $PC = \frac{1}{4}AC \Rightarrow PC = \frac{1}{2}OC$

In ΔDCO, E and P are the mid-points of DC and OC respectively.

$\therefore EP \parallel DO$

Also, in ΔCOB, P is the mid-point of OC and $PF \parallel DO \parallel BD$

Therefore, F is the mid-point of BC, F being EP produced.

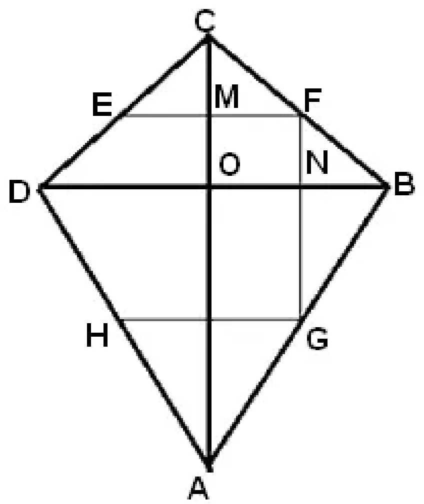
(ii) In ΔBCD, E and F are the mid-points of DC and BC respectively.

Also $EF \parallel BD$

Therefore, $EF = \frac{1}{2}BD$

$\Rightarrow \quad 2EF = BD$

Answer 22.



(i) Diagonals of a kite intersect at right angles

$\therefore \angle MON = 90^{\circ}$ ....(i)

In ΔBCD,

E and F are mid-points of CD and BC respectively.

Therefore, $EF \parallel DB$ and $EF = \frac{1}{2}DB$ ....(ii)

)°

3 and HG || DB (from (ii), EF||DB and EF||HG)

-point of DA.
through G and parallel to FE bisects DA

EF || DB $\Rightarrow$ MF || ON

$\therefore \angle MON + \angle MFN = 18($

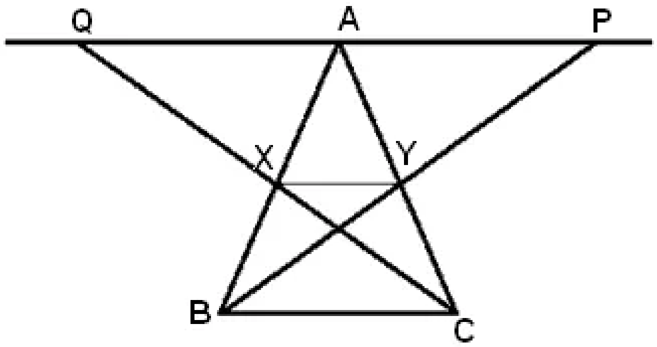
$\Rightarrow 90^\circ + \angle MFN = 180^\circ$
$\Rightarrow \angle MFN = 90^\circ$
$\Rightarrow \angle EFG = 90^\circ$

(ii) In Δ ABD,
G is the mid-point of AI

Therefore, HG||DB

Therefore, H is the mid
Hence, the line drawn t

Answer 23.



Join X and Y

In ΔABP,

X and Y are the mid-points of AB and AC respectively

Therefore, XY||BC

Since BC||AP
$\Rightarrow$XY||AP and XY||AQ

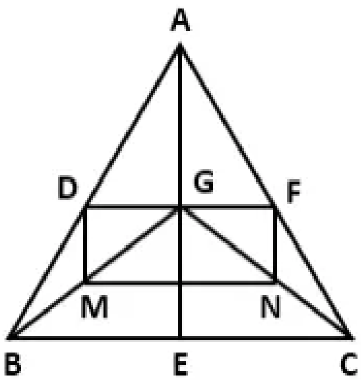
$\therefore XY = \frac{1}{2}AP........(i)$

$XY = \frac{1}{2}AQ..........(ii)$

From (i) and (ii)

$\Rightarrow \frac{1}{2}AP = \frac{1}{2}AQ$
$\Rightarrow AP = AQ$

Answer 24.



a. Since D and F are mid-points of AB and AC, by Mid-point theorem,

BC = 2DF

Now, BC = BE + EC

DF = DG + GF

But E is the mid-point of BC,

$\Rightarrow$ BE = EC ....(i)

Also, AG = GE ....(G is the mid-point of AE)

Consider ΔABE and ΔACE, by mid-point theorem,

BE = 2DG and EC = 2GF

$\Rightarrow$ 2DG = 2GF ....[From (i)]

$\Rightarrow$ DG = GF

Hence, AE and DF bisect each other.

b. Consider ΔABC and ΔGBC, by mid-point theorem,

2DF = BC and 2MN = BC

$\Rightarrow$ DF = MN ....(i)

Consider ΔABG and ΔACG, by mid-point theorem,

2DM = AG and 2FN = AG

$\Rightarrow$ DM = FN ....(ii)

From (i) and (ii), it is clear that DMNF is a parallelogram.