

Chapter 26. Trigonometrical Ratios

Ex 26.1

Answer 1.

$$(i) \sin A = \frac{12}{13}$$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{12}{13}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow \text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$$

$$\Rightarrow \text{Base} = \sqrt{(13)^2 - (12)^2} = \sqrt{169 - 144} = \sqrt{25} \\ = 5$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{5}{13}$$

$$\sec A = \frac{1}{\cos A} = \frac{13}{5}$$

$$\cot A = \frac{1}{\tan A} = \frac{5}{12}$$

$$\operatorname{cosec}A = \frac{1}{\sin A} = \frac{13}{12}$$

$$(ii) \cos B = \frac{4}{5}$$

$$\cos B = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow \text{Perpendicular} = \sqrt{(\text{Hypotenuse})^2 - (\text{Base})^2}$$

$$\begin{aligned} \Rightarrow \text{Perpendicular} &= \sqrt{(5)^2 - (4)^2} = \sqrt{25 - 16} = \sqrt{9} \\ &= 3 \end{aligned}$$

$$\sin B = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{5}$$

$$\tan B = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$$

$$\sec B = \frac{1}{\cos B} = \frac{5}{4}$$

$$\cot B = \frac{1}{\tan B} = \frac{4}{3}$$

$$\operatorname{cosec}B = \frac{1}{\sin B} = \frac{5}{3}$$

$$(iii) \cot A = \frac{1}{11}$$

$$\cot A = \frac{1}{\tan A} = \frac{\text{Base}}{\text{Perpendicular}}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\begin{aligned} (\text{Hypotenuse}) &= \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2} \\ &= \sqrt{(11)^2 + (1)^2} = \sqrt{121 + 1} = \sqrt{122} \end{aligned}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{1}{\sqrt{122}}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = 11$$

$$\sec A = \frac{1}{\cos A} = \sqrt{122}$$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{11}{\sqrt{122}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sqrt{122}}{11}$$

$$(iv) \operatorname{cosec} C = \frac{15}{11}$$

$$\operatorname{cosec} C = \frac{1}{\sin C} = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{15}{11}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow \text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$$

$$\Rightarrow \text{Base} = \sqrt{(15)^2 - (11)^2} = \sqrt{225 - 121} = \sqrt{104}$$

$$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{11}{15}$$

$$\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{104}}{15}$$

$$\tan C = \frac{\text{Perpendicular}}{\text{Base}} = \frac{11}{\sqrt{104}}$$

$$\sec C = \frac{1}{\cos C} = \frac{15}{\sqrt{104}}$$

$$\cot C = \frac{1}{\tan A} = \frac{\sqrt{104}}{11}$$

$$(iv) \operatorname{cosec} C = \frac{15}{11}$$

$$\operatorname{cosec} C = \frac{1}{\sin C} = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{15}{11}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow \text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$$

$$\Rightarrow \text{Base} = \sqrt{(15)^2 - (11)^2} = \sqrt{225 - 121} = \sqrt{104}$$

$$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{11}{15}$$

$$\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{104}}{15}$$

$$\tan C = \frac{\text{Perpendicular}}{\text{Base}} = \frac{11}{\sqrt{104}}$$

$$\sec C = \frac{1}{\cos C} = \frac{15}{\sqrt{104}}$$

$$\cot C = \frac{1}{\tan C} = \frac{\sqrt{104}}{11}$$

$$(v) \tan C = \frac{5}{12}$$

$$\tan C = \frac{\text{Perpendicular}}{\text{Base}} = \frac{5}{12}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\begin{aligned}(\text{Hypotenuse}) &= \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2} \\ &= \sqrt{(5)^2 + (12)^2} = \sqrt{25 + 144} = \sqrt{169} \\ &= 13\end{aligned}$$

$$\cot C = \frac{1}{\tan C} = \frac{12}{5}$$

$$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{5}{13}$$

$$\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{12}{13}$$

$$\sec C = \frac{1}{\cos C} = \frac{13}{12}$$

$$\text{cosec } C = \frac{1}{\sin C} = \frac{13}{5}$$

$$(vi) \sin B = \frac{\sqrt{3}}{2}$$

$$\sin B = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\sqrt{3}}{2}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow \text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$$

$$\Rightarrow \text{Base} = \sqrt{(2)^2 - (\sqrt{3})^2} = \sqrt{4 - 3} = \sqrt{1} \\ = 1$$

$$\cos B = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{1}{2}$$

$$\tan B = \frac{\text{Perpendicular}}{\text{Base}} = \sqrt{3}$$

$$\sec B = \frac{1}{\cos B} = 2$$

$$\cot B = \frac{1}{\tan B} = \frac{1}{\sqrt{3}}$$

$$\operatorname{cosec} B = \frac{1}{\sin A} = \frac{2}{\sqrt{3}}$$

$$\text{(vii) } \cos A = \frac{7}{25}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{7}{25}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow \text{Perpendicular} = \sqrt{(\text{Hypotenuse})^2 - (\text{Base})^2}$$

$$\Rightarrow \text{Perpendicular} = \sqrt{(25)^2 - (7)^2} = \sqrt{625 - 49} = \sqrt{576} \\ = 24$$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{24}{25}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{24}{7}$$

$$\sec A = \frac{1}{\cos A} = \frac{25}{7}$$

$$\cot A = \frac{1}{\tan A} = \frac{7}{24}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{25}{24}$$

$$\text{(viii) } \tan B = \frac{8}{15}$$

$$\tan B = \frac{\text{Perpendicular}}{\text{Base}} = \frac{8}{15}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\begin{aligned}(\text{Hypotenuse}) &= \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2} \\ &= \sqrt{(8)^2 + (15)^2} = \sqrt{64 + 225} = \sqrt{289} \\ &= 17\end{aligned}$$

$$\cot B = \frac{1}{\tan B} = \frac{15}{8}$$

$$\sin B = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{8}{17}$$

$$\cos B = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{15}{17}$$

$$\sec B = \frac{1}{\cos B} = \frac{17}{15}$$

$$\operatorname{cosec} B = \frac{1}{\sin B} = \frac{17}{8}$$

(ix) $\sec B = \frac{15}{12}$

$$\sec B = \frac{1}{\cos B} = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{15}{12}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow \text{Perpendicular} = \sqrt{(\text{Hypotenuse})^2 - (\text{Base})^2}$$

$$\begin{aligned}\Rightarrow \text{Perpendicular} &= \sqrt{(15)^2 - (12)^2} = \sqrt{225 - 144} = \sqrt{81} \\ &= 9\end{aligned}$$

$$\sin B = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{9}{15}$$

$$\tan B = \frac{\text{Perpendicular}}{\text{Base}} = \frac{9}{12}$$

$$\cot B = \frac{1}{\tan B} = \frac{12}{9}$$

$$\operatorname{cosec} B = \frac{1}{\sin B} = \frac{15}{9}$$

$$\cos B = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{12}{15}$$

$$(x) \quad \operatorname{cosec} C = \sqrt{10}$$

$$\operatorname{cosec} C = \frac{1}{\sin C} = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\sqrt{10}}{1}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow \text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$$

$$\begin{aligned} \Rightarrow \text{Base} &= \sqrt{(\sqrt{10})^2 - (1)^2} = \sqrt{10 - 1} = \sqrt{9} \\ &= 3 \end{aligned}$$

$$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{1}{\sqrt{10}}$$

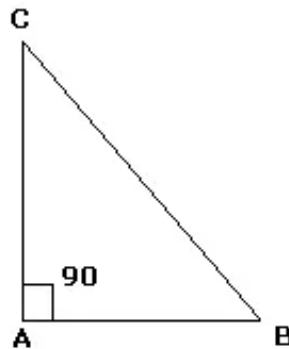
$$\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{3}{\sqrt{10}}$$

$$\tan C = \frac{\text{Perpendicular}}{\text{Base}} = \frac{1}{3}$$

$$\sec C = \frac{1}{\cos C} = \frac{\sqrt{10}}{3}$$

$$\cot C = \frac{1}{\tan A} = 3$$

Answer 2.



In $\triangle ABC$,

$$\begin{aligned}BC^2 &= AB^2 + AC^2 \\ \Rightarrow BC &= \sqrt{AB^2 + AC^2} \\ \Rightarrow BC &= \sqrt{5^2 + 12^2} \\ &= \sqrt{169} = 13\end{aligned}$$

$$AC = 12\text{units}$$

$$BC = 13\text{units}$$

$$AB = 5\text{units}$$

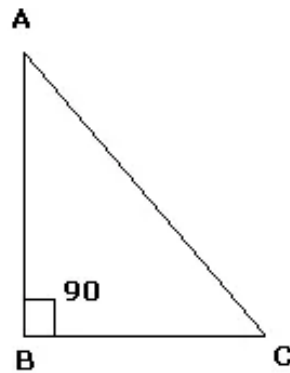
$$(i) \sin B = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AC}{BC} = \frac{12}{13}$$

$$\text{and } \cos B = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{BC} = \frac{5}{13}$$

$$(ii) \cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AC}{BC} = \frac{12}{13}$$

$$(iii) \tan B = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AC}{AB} = \frac{12}{5}$$

Answer 3.



In $\triangle ABC$,

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC = \sqrt{AB^2 + BC^2}$$

$$\Rightarrow AC = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} \\ = 13$$

$$AB = 12 \text{ units}$$

$$BC = 5 \text{ units}$$

$$AC = 13 \text{ units}$$

$$(i) \sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5}{13}$$

$$(ii) \tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{5}{12}$$

$$(iii) \cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5}{13}$$

$$(iv) \cot C = \frac{\text{Base}}{\text{Perpendicular}} = \frac{BC}{AB} = \frac{5}{12}$$

Answer 4.

$$\sin A = \frac{3}{5} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

By Pythagoras theorem, we have

$$\Rightarrow (\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow (\text{Base})^2 = (\text{Hypotenuse})^2 - (\text{Perpendicular})^2$$

$$\Rightarrow (\text{Base}) = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$$

$$\Rightarrow (\text{Base}) = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$$

Answer 5.

$$\cos B = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AB}$$

$$(AB)^2 = (AC)^2 + (BC)^2$$

$$\Rightarrow AC = \sqrt{(AB)^2 - (BC)^2}$$

$$\Rightarrow AC = \sqrt{3^2 - 1} = \sqrt{9 - 1} = 2\sqrt{2}$$

$$\sin A = \frac{BC}{AB} = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{1}{3}$$

$$\tan B = \frac{AC}{BC} = \frac{\text{Perpendicular}}{\text{Base}} = 2\sqrt{2}$$

$$\cot A = \frac{1}{\tan A} = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AC}{BC} = 2\sqrt{2}$$

Answer 6.

$$\sin \theta = \frac{8}{17} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\begin{aligned} \text{Base} &= \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2} \\ &= \sqrt{17^2 - 8^2} = \sqrt{225} = 15 \end{aligned}$$

$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{15}{17}$$

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{8}{15}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{17}{8}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{17}{15}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{15}{8}$$

Answer 7.

$$\tan A = 0.75 = \frac{75}{100} = \frac{3}{4} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\begin{aligned}\text{Hypotenuse} &= \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2} \\ &= \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} \\ &= 5\end{aligned}$$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{5} = 0.6$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5} = 0.8$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{5}{3} = 1.66$$

$$\sec A = \frac{1}{\cos A} = \frac{5}{4} = 1.25$$

$$\cot A = \frac{1}{\tan A} = \frac{4}{3} = 1.33$$

Answer 8.

$$\sin A = 0.8 = \frac{8}{10} = \frac{4}{5} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\begin{aligned}\text{Base} &= \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2} \\ &= \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3\end{aligned}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{3}{5} = 0.6$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{4}{3} = 1.33$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{5}{4} = 1.25$$

$$\sec A = \frac{1}{\cos A} = \frac{5}{3} = 1.66$$

$$\cot A = \frac{1}{\tan A} = \frac{3}{4} = 0.75$$

Answer 9.

$$8 \tan \theta = 15$$

$$\Rightarrow \tan \theta = \frac{15}{8} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\begin{aligned} \text{Hypotenuse} &= \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2} \\ &= \sqrt{15^2 + 8^2} \\ &= \sqrt{225 + 64} = \sqrt{289} \\ &= 17 \end{aligned}$$

$$(i) \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{15}{17}$$

$$(ii) \cot \theta = \frac{1}{\tan \theta} = \frac{8}{15}$$

$$\begin{aligned} (iii) \sin^2 \theta - \cot^2 \theta &= (\sin \theta + \cot \theta)(\sin \theta - \cot \theta) \\ &= \left(\frac{15}{17} + \frac{8}{15} \right) \left(\frac{15}{17} - \frac{8}{15} \right) \\ &= \left(\frac{225 + 136}{255} \right) \left(\frac{225 - 136}{255} \right) \\ &= \left(\frac{361}{255} \right) \left(\frac{89}{255} \right) = \frac{32129}{65025} \end{aligned}$$

Answer 14.

We are given that $BD : DC = 1 : 2$ as AD divides BC in the ratio $1 : 2$.

i.e $BD = x, DC = 2x \Rightarrow BC = 3x$

$$(i) \frac{\tan \angle BAC}{\tan \angle BAD} = \frac{\frac{BC}{AB}}{\frac{BD}{AB}} = \frac{BC}{BD} = \frac{3x}{x} = 3$$

$$(ii) \frac{\cot \angle BAC}{\cot \angle BAD} = \frac{\frac{AB}{BC}}{\frac{AB}{BD}} = \frac{BD}{BC} = \frac{x}{3x} = \frac{1}{3}$$

Answer 19.

As PS is the median on QR from P.

$$\therefore QS = SR \Rightarrow QR = 2QS$$

and RT divides PQ in the ratio 1:2

$$\therefore QT = x \text{ and } PT = 2x$$

$$\Rightarrow PQ = 3x$$

$$(i) \frac{\tan \angle PSQ}{\tan \angle PRQ} = \frac{\frac{PQ}{QS}}{\frac{PQ}{QR}} = \frac{PQ}{QS} \times \frac{QR}{PQ} = \frac{2QS}{QS} = 2$$

$$(ii) \frac{\tan \angle TSQ}{\tan \angle PRQ} = \frac{\frac{QT}{QS}}{\frac{PQ}{QR}} = \frac{QT}{QS} \times \frac{QR}{PQ} = \frac{x}{QS} \times \frac{2QS}{3x} = \frac{2}{3}$$

Answer 22.

$$24 \cos \theta = 7 \sin \theta$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{24}{7}$$

$$\Rightarrow \tan \theta = \frac{24}{7} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\begin{aligned} \text{Hypotenuse} &= \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2} \\ &= \sqrt{(24)^2 + (7)^2} \\ &= \sqrt{576 + 49} = \sqrt{625} = 25 \end{aligned}$$

$$\begin{aligned} \sin \theta + \cos \theta &= \frac{\text{Perpendicular}}{\text{Hypotenuse}} + \frac{\text{Base}}{\text{Hypotenuse}} \\ &= \frac{24}{25} + \frac{7}{25} = \frac{24+7}{25} = \frac{31}{25} \end{aligned}$$

Answer 24.

$$8 \tan A = 15$$

$$\Rightarrow \tan A = \frac{15}{8} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\begin{aligned} \text{Hypotenuse} &= \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2} \\ &= \sqrt{(15)^2 + (8)^2} \\ &= \sqrt{225 + 64} = \sqrt{289} = 17 \end{aligned}$$

$$\begin{aligned} \sin A - \cos A &= \frac{\text{Perpendicular}}{\text{Hypotenuse}} - \frac{\text{Base}}{\text{Hypotenuse}} \\ &= \frac{15}{17} - \frac{8}{17} = \frac{15-8}{17} \end{aligned}$$

$$\sin A - \cos A = \frac{7}{17}$$

Answer 25.

$$\begin{aligned}
3\cos\theta - 4\sin\theta &= 2\cos\theta + \sin\theta \\
\Rightarrow 3\cos\theta - 2\cos\theta &= \sin\theta + 4\sin\theta \\
\Rightarrow \cos\theta &= 5\sin\theta \\
\Rightarrow \frac{\sin\theta}{\cos\theta} &= \frac{1}{5} \\
\Rightarrow \tan\theta &= \frac{1}{5}
\end{aligned}$$

Answer 26.

$$\begin{aligned}
\text{If } 5\cos\theta &= 3 \\
\Rightarrow \cos\theta &= \frac{3}{5} = \frac{\text{Base}}{\text{Hypotenuse}} \\
\text{Perpendicular} &= \sqrt{(\text{Hypotenuse})^2 - (\text{Base})^2} \\
&= \sqrt{(5)^2 - (3)^2} \\
&= \sqrt{25 - 9} = \sqrt{16} \\
&= 4 \\
\sin\theta &= \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{4}{5} \\
\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta} &= \frac{4 \times \frac{3}{5} - \frac{4}{5}}{2 \times \frac{3}{5} + \frac{4}{5}} = \frac{\frac{12}{5} - \frac{4}{5}}{\frac{6}{5} + \frac{4}{5}} = \frac{\frac{8}{5}}{\frac{10}{5}} = \frac{4}{5}
\end{aligned}$$

Answer 28.

$$\begin{aligned}
5\tan\theta &= 12 \\
\Rightarrow \tan\theta &= \frac{12}{5} = \frac{\text{Perpendicular}}{\text{Base}} \\
\text{Hypotenuse} &= \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2} \\
&= \sqrt{(12)^2 + (5)^2} \\
&= \sqrt{144 + 25} = \sqrt{169} = 13 \\
\sin\theta &= \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{12}{13}, \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{5}{13} \\
\Rightarrow \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} &= \frac{2 \times \frac{12}{13} - 3 \times \frac{5}{13}}{4 \times \frac{12}{13} - 9 \times \frac{5}{13}} = \frac{24 - 15}{48 - 45} = \frac{9}{3} = 3
\end{aligned}$$

Answer 30.

$$\cot \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \cot \theta = \frac{1}{\tan \theta} = \frac{1}{\sqrt{3}} = \frac{\text{Base}}{\text{Perpendicular}}$$

$$\begin{aligned} \text{Hypotenuse} &= \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2} \\ &= \sqrt{(\sqrt{3})^2 + 1} = \sqrt{3+1} = 2 \end{aligned}$$

$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{1}{2},$$

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\sqrt{3}}{2}$$

$$\text{To show: } \frac{1 - \cos^2 \theta}{2 - \sin^2 \theta} = \frac{3}{5}$$

$$\frac{1 - \cos^2 \theta}{2 - \sin^2 \theta} = \frac{1 - (\cos \theta)^2}{2 - (\sin \theta)^2} = \frac{1 - \frac{1}{4}}{2 - \frac{3}{4}} = \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5}$$

Answer 31.

$$\operatorname{cosec} \theta = 1 \frac{9}{20} = \frac{29}{20}$$

$$\sin \theta = \frac{1}{\operatorname{cosec} \theta} = \frac{20}{29} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\begin{aligned} \text{Base} &= \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2} \\ &= \sqrt{(29)^2 - (20)^2} = \sqrt{841 - 400} \\ &= \sqrt{441} = 21 \end{aligned}$$

$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{21}{29}$$

$$\text{To show: } \frac{1 - \sin \theta + \cos \theta}{1 + \sin \theta + \cos \theta} = \frac{3}{7}$$

$$\begin{aligned} \frac{1 - \sin \theta + \cos \theta}{1 + \sin \theta + \cos \theta} &= \frac{1 - \frac{20}{29} + \frac{21}{29}}{1 + \frac{20}{29} + \frac{21}{29}} \\ &= \frac{29 - 20 + 21}{29 + 20 + 21} \\ &= \frac{30}{70} = \frac{3}{7} \end{aligned}$$

Answer 32.

$$b \tan \theta = a$$

$$\Rightarrow \tan \theta = \frac{a}{b}$$

Consider $\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$

Dividing the numerator and denominator by $\cos \theta$, we get

$$\begin{aligned} \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} &= \frac{1 + \frac{\sin \theta}{\cos \theta}}{1 - \frac{\sin \theta}{\cos \theta}} = \frac{1 + \tan \theta}{1 - \tan \theta} \\ &= \frac{1 + \frac{a}{b}}{1 - \frac{a}{b}} = \frac{\frac{b+a}{b}}{\frac{b-a}{b}} = \frac{(b+a)}{(b-a)} \end{aligned}$$

Answer 33.

$$a \cot \theta = b$$

$$\Rightarrow \cot \theta = \frac{b}{a}$$

$$\Rightarrow \tan \theta = \frac{1}{\cot \theta} = \frac{a}{b}$$

To prove: $\frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta} = \frac{a^2 - b^2}{a^2 + b^2}$

Consider $\frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta}$

Dividing the numerator and denominator by $\cos \theta$, we get

$$\begin{aligned} \frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta} &= \frac{a \frac{\sin \theta}{\cos \theta} - b}{a \frac{\sin \theta}{\cos \theta} + b} = \frac{a \tan \theta - b}{a \tan \theta + b} \\ &= \frac{a \times \frac{a}{b} - b}{a \times \frac{a}{b} + b} = \frac{\frac{a^2 - b^2}{b}}{\frac{a^2 + b^2}{b}} = \frac{a^2 - b^2}{a^2 + b^2} \end{aligned}$$

Answer 34.

$$\cot \theta = \sqrt{7}$$

$$\Rightarrow \frac{\cos \theta}{\sin \theta} = \sqrt{7}$$

$$\Rightarrow \frac{\text{base}}{\text{hypotenuse}} \times \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{\sqrt{7}}{1}$$

$$\Rightarrow \frac{\text{base}}{\text{perpendicular}} = \frac{\sqrt{7}}{1}$$

$$\begin{aligned} \text{Hypotenuse} &= \sqrt{(\text{perpendicular})^2 + (\text{Base})^2} \\ &= \sqrt{1 + 7} = 2\sqrt{2} \end{aligned}$$

$$\text{To show: } \frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta} = \frac{3}{4}$$

$$\begin{aligned} \frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta} &= \frac{\left(\frac{\text{hypotenuse}}{\text{perpendicular}}\right)^2 - \left(\frac{\text{hypotenuse}}{\text{base}}\right)^2}{\left(\frac{\text{hypotenuse}}{\text{perpendicular}}\right)^2 + \left(\frac{\text{hypotenuse}}{\text{base}}\right)^2} \\ &= \frac{\left(\frac{2\sqrt{2}}{1}\right)^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{\left(\frac{2\sqrt{2}}{1}\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2} = \frac{\frac{8}{1} - \frac{8}{7}}{\frac{8}{1} + \frac{8}{7}} = \frac{\frac{56 - 8}{7}}{\frac{56 + 8}{7}} = \frac{48}{64} \\ &= \frac{3}{4} \end{aligned}$$

Answer 35.

$$12 \operatorname{cosec} \theta = 13$$

$$\Rightarrow \operatorname{cosec} \theta = \frac{13}{12}$$

$$\Rightarrow \sin \theta = \frac{12}{13} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\begin{aligned} \Rightarrow \text{Base} &= \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2} \\ &= \sqrt{(13)^2 - (12)^2} = \sqrt{169 - 144} = \sqrt{25} = 5 \end{aligned}$$

$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{5}{13}$$

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{12}{5}$$

$$\text{Now, } \frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cos \theta} \times \frac{1}{\tan^2 \theta}$$

$$= \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2 \left(\frac{12}{13}\right) \left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

$$= \frac{\frac{144}{169} - \frac{25}{169}}{\frac{120}{169}} \times \frac{25}{144}$$

$$= \frac{119}{120} \times \frac{25}{144} = \frac{595}{3456}$$

Answer 36.

$$\cot \theta = \frac{13}{12}$$

$$\Rightarrow \frac{\cos \theta}{\sin \theta} = \frac{13}{12}$$

$$\Rightarrow \frac{\text{base}}{\text{hypotenuse}} \times \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{13}{12}$$

$$\Rightarrow \frac{\text{base}}{\text{perpendicular}} = \frac{13}{12}$$

$$\begin{aligned} \text{Hypotenuse} &= \sqrt{(\text{perpendicular})^2 + (\text{Base})^2} \\ &= \sqrt{(12)^2 + (13)^2} = \sqrt{144 + 169} = \sqrt{313} \end{aligned}$$

$$\begin{aligned} \frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} &= \frac{2 \times \frac{12}{\sqrt{313}} \times \frac{13}{\sqrt{313}}}{\left(\frac{13}{\sqrt{313}}\right)^2 - \left(\frac{12}{\sqrt{313}}\right)^2} = \frac{\frac{312}{313}}{\frac{169}{313} - \frac{144}{313}} = \frac{\frac{312}{313}}{\frac{25}{313}} \\ &= \frac{312}{25} \end{aligned}$$

Answer 37.

$$\sec A = \frac{5}{4}$$

$$\Rightarrow \cos A = \frac{4}{5} = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\begin{aligned} \text{Perpendicular} &= \sqrt{(\text{Hypotenuse})^2 - (\text{Base})^2} \\ &= \sqrt{25 - 16} = \sqrt{9} = 3 \end{aligned}$$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{5}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$$

$$\text{To show: } \frac{3 \sin A - 4 \sin^3 A}{4 \cos^3 A - 3 \cos A} = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

$$\begin{aligned} \text{L.H.S} &= \frac{3 \sin A - 4 \sin^3 A}{4 \cos^3 A - 3 \cos A} = \frac{3\left(\frac{3}{5}\right) - 4\left(\frac{3}{5}\right)^3}{4\left(\frac{4}{5}\right)^3 - 3\left(\frac{4}{5}\right)} = \frac{\frac{9}{5} - \frac{108}{125}}{\frac{256}{125} - \frac{12}{5}} \end{aligned}$$

$$= \frac{\frac{225 - 108}{125}}{\frac{256 - 300}{125}} = \frac{117}{-44}$$

$$\begin{aligned} \text{R.H.S} &= \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A} = \frac{3\left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^3}{1 - 3\left(\frac{3}{4}\right)^2} = \frac{\frac{9}{4} - \frac{27}{64}}{1 - \frac{27}{16}} = \frac{\frac{144 - 27}{64}}{\frac{16 - 27}{16}} \\ &= \frac{117}{64} \times \frac{16}{-11} = \frac{117}{4} \times \frac{1}{-11} = \frac{-117}{44} \end{aligned}$$

$\Rightarrow \text{L.H.S} = \text{R.H.S}$

Answer 38.

$$\sin \theta = \frac{3}{4}$$

$$\frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{4}$$

$$\begin{aligned} \text{Base} &= \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2} \\ &= \sqrt{16 - 9} = \sqrt{7} \end{aligned}$$

$$\operatorname{cosec} \theta = \frac{4}{3}$$

$$\cot \theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{\sqrt{7}}{3}$$

$$\sec \theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{4}{\sqrt{7}}$$

$$\text{To prove } \sqrt{\frac{\operatorname{cosec}^2 \theta - \cot^2 \theta}{\sec^2 \theta - 1}} = \frac{\sqrt{7}}{3}$$

$$\begin{aligned} \sqrt{\frac{\operatorname{cosec}^2 \theta - \cot^2 \theta}{\sec^2 \theta - 1}} &= \sqrt{\frac{\left(\frac{4}{3}\right)^2 - \left(\frac{\sqrt{7}}{3}\right)^2}{\left(\frac{4}{\sqrt{7}}\right)^2 - 1}} \\ &= \sqrt{\frac{\frac{16}{9} - \frac{7}{9}}{\frac{16}{7} - 1}} = \sqrt{\frac{\frac{16-7}{9}}{\frac{16-7}{7}}} = \sqrt{\frac{9}{9}} \\ &= \sqrt{\frac{1}{9}} = \sqrt{\frac{7}{9}} = \frac{\sqrt{7}}{3} \end{aligned}$$

Answer 39.

$$\sec A = \frac{17}{8}$$

$$\Rightarrow \cos A = \frac{8}{17} = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\begin{aligned} \text{Perpendicular} &= \sqrt{(\text{Hypotenuse})^2 - (\text{Base})^2} \\ &= \sqrt{(17)^2 - (8)^2} = \sqrt{289 - 64} = \sqrt{225} = 15 \end{aligned}$$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{15}{17}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{15}{8}$$

$$\text{To prove: } \frac{3 - 4\sin^2 A}{4\cos^2 A - 3} = \frac{3 - \tan^2 A}{1 - 3\tan^2 A}$$

$$\text{L.H.S} = \frac{3 - 4\sin^2 A}{4\cos^2 A - 3} = \frac{3 - 4\left(\frac{15}{17}\right)^2}{4\left(\frac{8}{17}\right)^2 - 3} = \frac{3 - \frac{900}{289}}{\frac{256}{289} - 3} = \frac{\frac{867 - 900}{289}}{\frac{256 - 867}{289}} = \frac{-33}{-611} = \frac{33}{611}$$

$$\text{R.H.S} = \frac{3 - \tan^2 A}{1 - 3\tan^2 A} = \frac{3 - \left(\frac{15}{8}\right)^2}{1 - 3\left(\frac{15}{8}\right)^2} = \frac{3 - \frac{225}{64}}{1 - \frac{675}{64}} = \frac{\frac{192 - 225}{64}}{\frac{64 - 675}{64}} = \frac{-33}{-611} = \frac{33}{611}$$

$$\Rightarrow \text{L.H.S} = \text{R.H.S}$$

Answer 40.

$$3\tan\theta = 4$$

$$\Rightarrow \tan\theta = \frac{4}{3} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\begin{aligned} \text{Hypotenuse} &= \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2} \\ &= \sqrt{(4)^2 + (3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5 \end{aligned}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{5}{3}$$

$$\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{5}{4}$$

$$\text{To prove: } \frac{\sqrt{\sec\theta - \operatorname{cosec}\theta}}{\sqrt{\sec\theta + \operatorname{cosec}\theta}} = \frac{1}{\sqrt{7}}$$

$$\begin{aligned} \frac{\sqrt{\sec\theta - \operatorname{cosec}\theta}}{\sqrt{\sec\theta + \operatorname{cosec}\theta}} &= \frac{\sqrt{\frac{5}{3} - \frac{5}{4}}}{\sqrt{\frac{5}{3} + \frac{5}{4}}} = \frac{\sqrt{\frac{20 - 15}{12}}}{\sqrt{\frac{20 + 15}{12}}} = \frac{\sqrt{\frac{5}{12}}}{\sqrt{\frac{35}{12}}} = \frac{\sqrt{5}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{35}} = \frac{\sqrt{5}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{5} \times \sqrt{7}} \\ &= \frac{1}{\sqrt{7}} \end{aligned}$$

Answer 41.

$$\tan \theta = \frac{m}{n} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\begin{aligned}\text{Hypotenuse} &= \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2} \\ &= \sqrt{m^2 + n^2}\end{aligned}$$

$$\sin \theta = \left(\frac{m}{\sqrt{m^2 + n^2}} \right)$$

$$\cos \theta = \left(\frac{n}{\sqrt{m^2 + n^2}} \right)$$

$$\text{To show: } \frac{m \sin \theta - n \cos \theta}{m \sin \theta + n \cos \theta} = \frac{m^2 - n^2}{m^2 + n^2}.$$

$$\frac{m \sin \theta - n \cos \theta}{m \sin \theta + n \cos \theta} = \frac{m \left(\frac{m}{\sqrt{m^2 + n^2}} \right) - n \left(\frac{n}{\sqrt{m^2 + n^2}} \right)}{m \left(\frac{m}{\sqrt{m^2 + n^2}} \right) + n \left(\frac{n}{\sqrt{m^2 + n^2}} \right)}$$

$$\begin{aligned}&= \frac{\frac{m^2 - n^2}{\sqrt{m^2 + n^2}}}{\frac{m^2 + n^2}{\sqrt{m^2 + n^2}}} = \frac{m^2 - n^2}{\sqrt{m^2 + n^2}} \times \frac{\sqrt{m^2 + n^2}}{m^2 + n^2} \\ &= \frac{m^2 - n^2}{m^2 + n^2}\end{aligned}$$