

Pythagoras Theorem

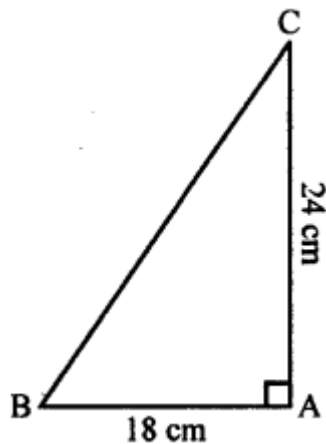
EXERCISE 16

Question 1.

Triangle ABC is right-angled at vertex A. Calculate the length of BC, if AB = 18 cm and AC = 24 cm.

Solution:

Given : $\triangle ABC$ right angled at A and AB = 18 cm, AC = 24 cm.



To find : Length of BC.

According to Pythagoras Theorem,

$$BC^2 = AB^2 + AC^2$$

$$= 18^2 + 24^2 = 324 + 576 = 900$$

$$\therefore BC = \sqrt{900} = \sqrt{30 \times 30} = 30 \text{ cm}$$

Question 2.

Triangle XYZ is right-angled at vertex Z. Calculate the length of YZ, if XY = 13 cm and XZ = 12 cm.

Solution:

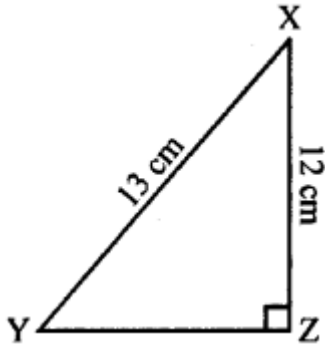
Given : $\triangle XYZ$ right angled at Z and XY = 13 cm, XZ = 12 cm.

To find : Length of YZ.

According to Pythagoras Theorem,

$$XY^2 = XZ^2 + YZ^2$$

$$13^2 = 12^2 + YZ^2$$



$$169 = 144 + YZ^2$$

$$169 - 144 = YZ^2$$

$$25 = YZ^2$$

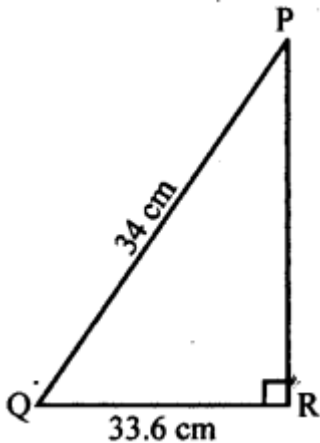
$$\therefore YZ = \sqrt{25} \text{ cm } \sqrt{5 \times 5} = 5 \text{ cm}$$

Question 3.

Triangle PQR is right-angled at vertex R. Calculate the length of PR, if: PQ = 34 cm and QR = 33.6 cm.

Solution:

Given : $\triangle PQR$ right angled at R and PQ = 34 cm, QR = 33.6 cm.



To find : Length of PR.

According to Pythagoras Theorem,

$$PR^2 + QR^2 = PQ^2$$

$$PR^2 + 33.6^2 = 34^2$$

$$PR^2 + 1128.96 = 1156$$

$$PR^2 = 1156 - 1128.96$$

$$\therefore PR = \sqrt{27.04} = 5.2 \text{ cm}$$

Question 4.

The sides of a certain triangle are given below. Find, which of them is right-triangle

(i) 16 cm, 20 cm and 12 cm

(ii) 6 m, 9 m and 13 m

Solution:

(i) 16 cm, 20 cm and 12 cm

The given triangle will be a right-angled triangle if square of its largest side is equal to the sum of the squares on the other two sides.

$$\text{i.e., If } (20)^2 = (16)^2 + (12)^2$$

$$(20)^2 = (16)^2 + (12)^2$$

$$400 = 256 + 144$$

$$400 = 400$$

So, the given triangle is right angled.

(ii) 6 m, 9 m and 13 m

The given triangle will be a right-angled triangle if square of its largest side is equal to the sum of the squares on the other two sides.

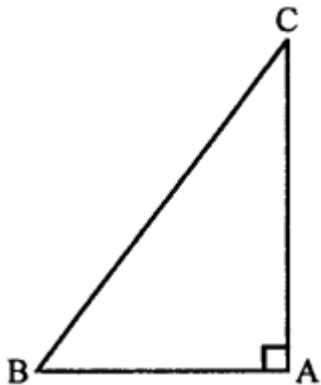
$$\text{i.e., If } (13)^2 = (9)^2 + (6)^2$$

$$169 = 81 + 36 \quad 169 \neq 117$$

So, the given triangle is not right angled.

Question 5.

In the given figure, angle $BAC = 90^\circ$, $AC = 400$ m and $AB = 300$ m. Find the length of BC .

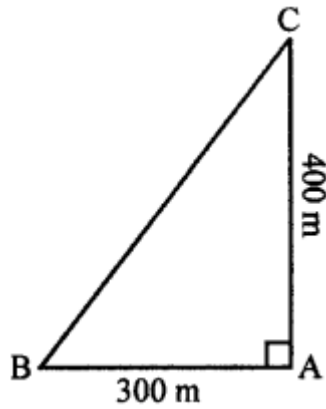


Solution:

$$AC = 400 \text{ m}$$

$$AB = 300 \text{ m}$$

$$BC = ?$$



According to Pythagoras Theorem,

$$BC^2 = AB^2 + AC^2$$

$$BC^2 = (300)^2 + (400)^2$$

$$BC^2 = 90000 + 160000$$

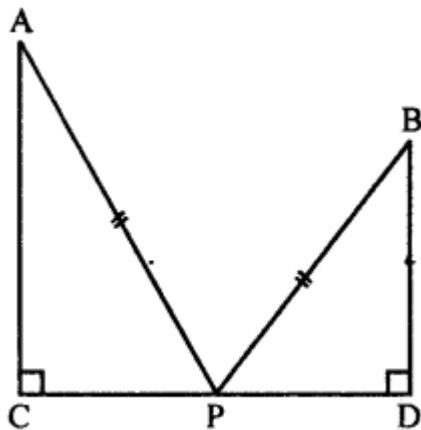
$$BC^2 = 250000$$

$$BC = \sqrt{250000} = 500 \text{ m}$$

Question 6.

In the given figure, angle $ACP = \angle BDP = 90^\circ$, $AC = 12 \text{ m}$, $BD = 9 \text{ m}$ and $PA = PB = 15 \text{ m}$. Find:

- (i) CP
- (ii) PD
- (iii) CD

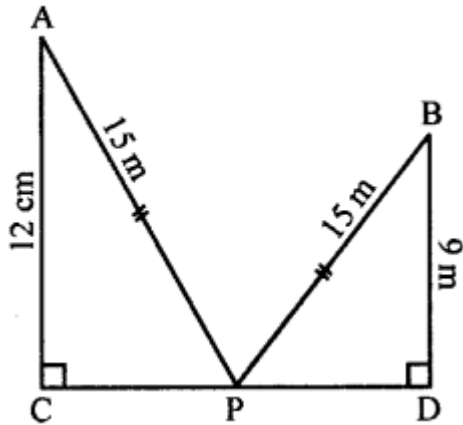


Solution:

Given : $AC = 12 \text{ m}$

$BD = 9 \text{ m}$

$PA = PB = 15 \text{ m}$



(i) In right angle triangle ACP

$$(AP)^2 = (AC)^2 + (CP)^2$$

$$15^2 = 12^2 + CP^2$$

$$225 = 144 + CP^2$$

$$225 - 144 = CP^2$$

$$81 = CP^2$$

$$\sqrt{81} = CP$$

$$\therefore CP = 9 \text{ m}$$

(ii) In right angle triangle BPD

$$(PB)^2 = (BD)^2 + (PD)^2$$

$$(15)^2 = (9)^2 + PD^2$$

$$225 = 81 + PD^2$$

$$225 - 81 = PD^2$$

$$144 = PD^2$$

$$\sqrt{144} = PD$$

$$\therefore PD = 12 \text{ m}$$

(iii) $CP = 9 \text{ m}$

$$PD = 12 \text{ m}$$

$$\therefore CD = CP + PD$$

$$= 9 + 12 = 21 \text{ m}$$

Question 7.

In triangle PQR, angle Q = 90° , find :

(i) PR, if PQ = 8 cm and QR = 6 cm

(ii) PQ, if PR = 34 cm and QR = 30 cm

Solution:

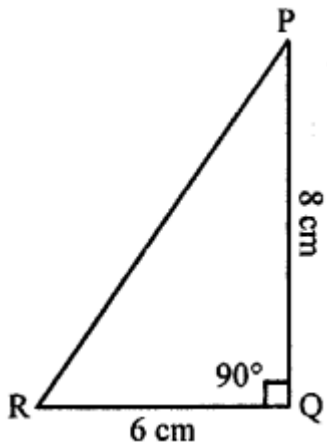
(i) Given:

$$PQ = 8 \text{ cm}$$

$$QR = 6 \text{ cm}$$

$$PR = ?$$

$$\angle PQR = 90^\circ$$



According to Pythagoras Theorem,

$$(PR)^2 = (PQ)^2 + (QR)^2$$

$$PR^2 = 8^2 + 6^2$$

$$PR^2 = 64 + 36$$

$$PR^2 = 100$$

$$\therefore PR = \sqrt{100} = 10 \text{ cm}$$

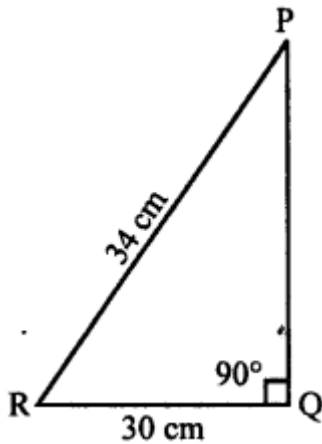
(ii) Given :

$$PR = 34 \text{ cm}$$

$$QR = 30 \text{ cm}$$

$$PQ = ?$$

$$\angle PQR = 90^\circ$$



According to Pythagoras Theorem,

$$(PR)^2 = (PQ)^2 + (QR)^2$$

$$(34)^2 = PQ^2 + (30)^2$$

$$1156 = PQ^2 + 900$$

$$1156 - 900 = PQ^2$$

$$256 = PQ^2$$

$$\therefore PQ = 16 \text{ cm}$$

Question 8.

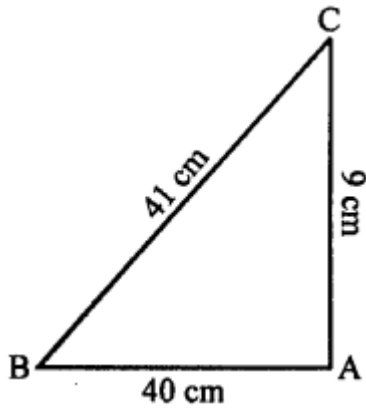
Show that the triangle ABC is a right-angled triangle; if:
AB = 9 cm, BC = 40 cm and AC = 41 cm

Solution:

AB = 9 cm

CB = 40 cm

AC = 41 cm



The given triangle will be a right angled triangle if square of its largest side is equal to the sum of the squares on the other two sides.

According to Pythagoras Theorem,

$$(AC)^2 = (BC)^2 + (AB)^2$$

$$(41)^2 = (40)^2 + (9)^2$$

$$1681 = 1600 + 81$$

$$1681 = 1681$$

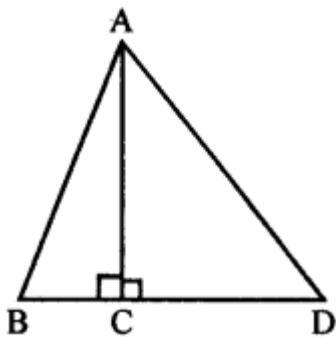
Hence, it is a right-angled triangle ABC.

Question 9.

In the given figure, angle ACB = 90° = angle ACD. If AB = 10 m, BC = 6 cm and AD = 17 cm, find :

(i) AC

(ii) CD



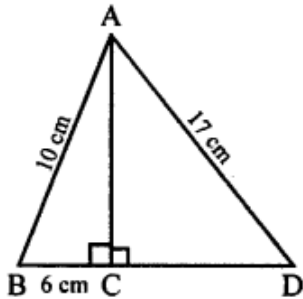
Solution:

Given:

$\triangle ABD$

$\angle ACB = \angle ACD = 90^\circ$

and $AB = 10$ cm, $BC = 6$ cm and $AD = 17$ cm



To find:

(i) Length of AC

(ii) Length of CD

Proof:

(i) In right-angled triangle ABC

$BC = 6$ cm, $AB = 10$ cm

According to Pythagoras Theorem,

$$AB^2 = AC^2 + BC^2$$

$$(10)^2 = (AC)^2 + (6)^2$$

$$100 = (AC)^2 + 36$$

$$AC^2 = 100 - 36 = 64$$

$$AC^2 = 64$$

$$\therefore AC = \sqrt{8 \times 8} = 8 \text{ cm}$$

(ii) In right-angle triangle ACD

$AD = 17$ cm, $AC = 8$ cm

According to Pythagoras Theorem,

$$(AD)^2 = (AC)^2 + (CD)^2$$

$$(17)^2 = (8)^2 + (CD)^2$$

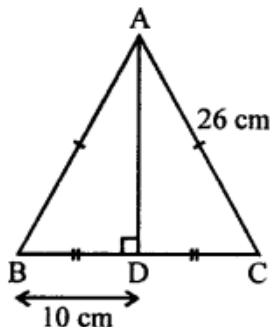
$$289 - 64 = CD^2$$

$$225 = CD^2$$

$$CD = \sqrt{15 \times 15} = 15 \text{ cm}$$

Question 10.

In the given figure, angle $ADB = 90^\circ$, $AC = AB = 26$ cm and $BD = DC$. If the length of $AD = 24$ cm; find the length of BC.



Solution:

Given:

$\triangle ABC$

$\angle ADB = 90^\circ$ and $AC = AB = 26$ cm

$AD = 24$ cm

To find : Length of BC In right angled $\triangle ADC$

$AB = 26$ cm, $AD = 24$ cm

According to Pythagoras Theorem,

$$(AC)^2 = (AD)^2 + (DC)^2$$

$$(26)^2 = (24)^2 + (DC)^2$$

$$676 = 576 + (DC)^2$$

$$\Rightarrow (DC)^2 = 100$$

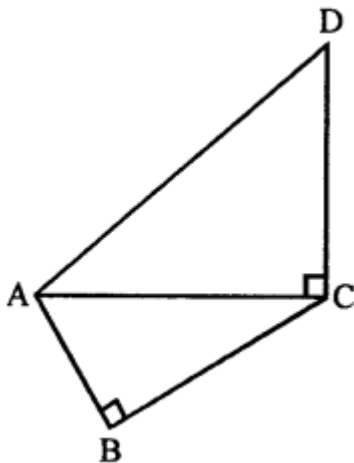
$$\Rightarrow DC = \sqrt{100} = 10 \text{ cm}$$

\therefore Length of BC = BD + DC

$$= 10 + 10 = 20 \text{ cm}$$

Question 11.

In the given figure, $AD = 13$ cm, $BC = 12$ cm, $AB = 3$ cm and angle $ACD =$ angle $ABC = 90^\circ$. Find the length of DC.



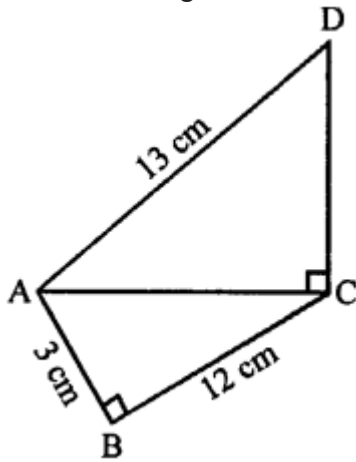
Solution:

Given :

$\triangle ACD = \triangle ABC = 90^\circ$

and $AD = 13$ cm, $BC = 12$ cm, $AB = 3$ cm

To find : Length of DC.



(i) In right angled $\triangle ABC$
 $AB = 3$ cm, $BC = 12$ cm
According to Pythagoras Theorem,
 $(AC)^2 = (AB)^2 + (BC)^2$
 $(AC)^2 = (3)^2 + (12)^2$
 $(AC) = \sqrt{9 + 144} = \sqrt{153}$ cm

(ii) In right angled triangle ACD
 $AD = 13$ cm, $AC = \sqrt{153}$
According to Pythagoras Theorem,
 $DC^2 = AD^2 - AC^2$
 $DC^2 = 169 - 153$
 $DC = \sqrt{16} = 4$ cm
 \therefore Length of DC is 4 cm

Question 12.

A ladder, 6.5 m long, rests against a vertical wall. If the foot of the ladder is 2.5 m from the foot of the wall, find upto how much height does the ladder reach?

Solution:

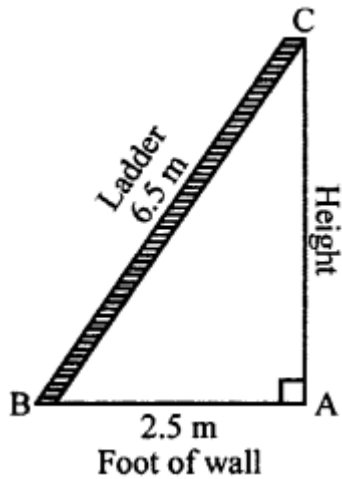
Given :

Length of ladder = 6.5 m

Length of foot of the wall = 2.5 m

To find : Height AC According to Pythagoras Theorem,

$$(BC)^2 = (AB)^2 + (AC)^2$$



$$(6.5)^2 = (2.5)^2 + (AC)^2$$

$$42.25 = 6.25 + AC^2$$

$$AC^2 = 42.25 - 6.25 = 36 \text{ m}$$

$$AC = \sqrt{6 \times 6} = 6 \text{ m}$$

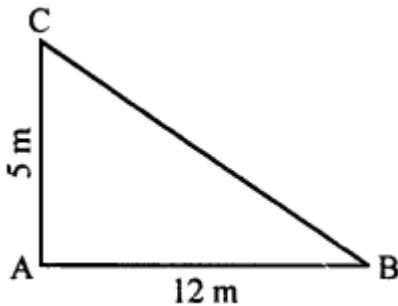
$$\therefore \text{Height of wall} = 6 \text{ m}$$

Question 13.

A boy first goes 5 m due north and then 12 m due east. Find the distance between the initial and the final position of the boy.

Solution:

Given : Direction of north = 5 m i.e. AC Direction of east = 12 m i.e. AB



To find: BC

According to Pythagoras Theorem,

In right angled AABC

$$(BC)^2 = (AC)^2 + (AB)^2$$

$$(BC)^2 = (5)^2 + (12)^2$$

$$(BC)^2 = 25 + 144$$

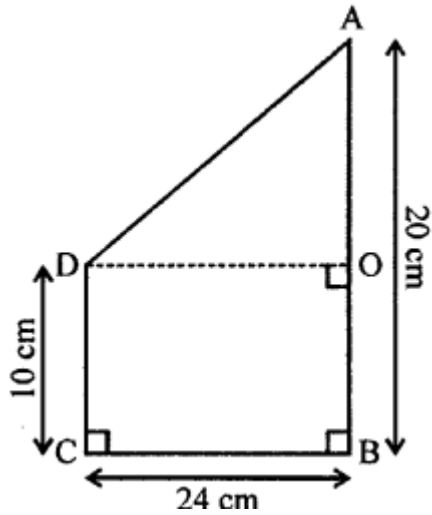
$$(BC)^2 = 25 + 144$$

$$(BC)^2 = 169$$

$$\therefore BC = \sqrt{169} = \sqrt{13 \times 13} = 13 \text{ m}$$

Question 14.

Use the information given in the figure to find the length AD.



Solution:

Given :

$$AB = 20 \text{ cm}$$

$$\therefore AO = \frac{AB}{2} = \frac{20}{2} = 10 \text{ cm}$$

$$BC = OD = 24 \text{ cm}$$

To find : Length of AD

In right angled triangle

$$AOD \quad (AD)^2 = (AO)^2 + (OD)^2$$

$$(AD)^2 = (10)^2 + (24)^2$$

$$(AD)^2 = 100 + 576$$

$$(AD)^2 = 676$$

$$\therefore AD = \sqrt{26 \times 26}$$

$$AD = 26 \text{ cm}$$